1. (36 points, 6 points apiece) Find the following. If an expression is undefined, say so.
(a) $\sum_{n=2}^{\infty} 5^{-n}$.
(b) $\sum_{n=1}^{\infty} (2^n + 2^{-n})$.
(c) The set of all real numbers $p$ such that $\sum_{n=2}^{\infty} n^{-1} (\ln n)^p$ converges.
(d) The Maclaurin series for $2^x$.
(e) The Taylor series for $1/x^2$ centered at $x = 1$.
(f) The solution to the differential equation $xy' = (x+1)y$ satisfying the initial condition $y(1) = 1$.

2. (16 points) Let $a$ and $b$ be real numbers. Prove that $\sum_{n=1}^{\infty} \left( \frac{a}{n} + \frac{b}{n+1} \right)$ converges if and only if $a + b = 0$.

3. (30 points, 6 points apiece) For each of the items listed below, give either an example, or a brief reason why no example exists. (If you give an example, you are not asked to show that it has the asserted property.)
(a) A power series $\sum_{n=0}^{\infty} a_n (x-1)^n$ with radius of convergence 3.
(b) A power series $\sum_{n=0}^{\infty} a_n x^n$ which converges for all $x \geq -1$ and no other $x$.
(c) A power series $\sum_{n=0}^{\infty} a_n (x-2)^n$ which converges for all real numbers $x$.
(d) A series $\sum_{n=1}^{\infty} a_n$ which is convergent but not absolutely convergent.
(e) Two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that $a_n \geq b_n$ for all $n$, and $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} b_n$ diverges.

4. (18 points) (a) (7 points) Find the first three terms (i.e., the constant, linear, and square terms) of the Taylor series for $\ln x$ centered at $x = 2$.
(b) (11 points) Prove using the formula for the remainder ("Taylor's Formula") that for all $x$ in the interval $[1.5, 2.5]$, the sum of the above three terms approximates $\ln x$ to within $1/81$. 