450 points total. The first 10 questions are Multiple Choice, worth 10 points each. For each question mark an × in the most correct place in the grid below.

No partial credit for 1-10.

Questions 16 through 21 are not multiple choice.

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GSI's only

MC  
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17  
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TOTAL
1. Which of the following is most correct concerning the function \( y = f(x) \), which is differentiable at \( a \).
   
   a) \( f'(a) = \lim_{\delta \to 0} \frac{f(a + \delta) - f(a)}{\delta} \)
   
   b) \( f'(a) = \lim_{\delta \to 0} \frac{f(a + \frac{\delta}{2}) - f(a)}{\delta} \)
   
   c) \( f'(a) = \lim_{\delta \to \frac{1}{2}} \frac{f(a + \delta) - f(a)}{(\frac{1}{2})} \)
   
   d) \( f'(a) = \lim_{\delta \to a} \frac{f(a) - f(a)}{\delta} \)
   
   e) \( f'(a) \) is the limiting slope of the straight line obtained by zooming in more and more on the graph of \( y = f(x) \) at \( x = a \).

2. Which of the following functions (for \( x \neq 0 \)) could have the following curve as its graph?
   
   a) \( y = x \ln |x| \)
   
   b) \( y = e^{-\frac{1}{x}} \)
   
   c) \( y = xe^x \)
   
   d) \( y = xe^{-x} \)
   
   e) \( y = -xe^{-x} \)

3. Newton’s method used to solve the equation \( \frac{1}{x} - a = 0 \) yields the following sequence of approximations
   
   a) \( x_{n+1} = 3x_n - ax_n^2 \)
   
   b) \( x_{n+1} = 2x_n - ax_n^2 \)
   
   c) \( x_{n+1} = \frac{1}{x_n} - a \)
   
   d) \( x_{n+1} = x_n - \frac{1}{a} \)
   
   e) \( x_{n+1} = ax_n \)
4. If the graph of $f'$ is

which of the following could be the graph of $f$?
5. Which of the following best represents the graph of the function \( f(x) = \frac{x^3 + 1}{x^2 + 1} \)?
6. Let \( f \) and \( g \) be two functions differentiable for all \( x \) and suppose that \( f(a) = g(a) \) and \( f''(x) > g''(x) + \frac{1}{2} \) for all \( x > a \). Which of the following can we conclude?

a) \( f(x) > g(x) \) for all \( x > a \)

b) \( f(x) \geq g(x) \) for all \( x > a \)

c) \( f(x) > g(x) \) for all \( x > c \) for some sufficiently large \( c \)

d) \( f'(x) > g'(x) \) for all \( x > a \)

e) \( f'(x) \geq g'(x) \) for all \( x > a \)

7. Which of the following functions is continuous?

a) \( f(x) = \begin{cases} \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \)

b) \( f(x) = \begin{cases} \frac{1}{x} \sin \frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases} \)

c) \( f(x) = \begin{cases} \sin \frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases} \)

d) \( f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \)

e) \( f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases} \)

8. A ladder 10 ft long is leaning against a vertical wall. It starts to slide. When the point of contact of the ladder with the wall is 6 ft from the base of the wall that point of contact is moving down at 2 ft/sec. How fast is the point where the ladder touches the ground moving at that time?

a) 1\( \frac{1}{2} \) ft/sec

b) 2 ft/sec

c) 2\( \frac{1}{2} \) ft/sec

d) 2\( \frac{1}{2} \) ft/sec

e) 2\( \frac{2}{3} \) ft/sec
9. The maximum value of \( y = x^3 - 6x^2 + 9x \) on the interval \([-2,0]\) is 
   a) 0  
   b) 4  
   c) 14  
   d) 16  
   e) It doesn't attain its maximum.

10. Which of the following best describes a solid whose volume is given by the formula \( \int_0^2 2\pi y(4-y^2)dy \) ?
   a) The solid obtained by rotating the region in the first quadrant bounded by \( x = 4-y^2 \) and the y-axis about the x axis.
   b) The solid obtained by rotating the region in the first quadrant bounded by \( x = y^2 \) and the y-axis about the x axis.
   c) The solid obtained by rotating the region in the first quadrant bounded by \( x = y^2 \) and the y-axis about the y axis.
   d) The solid obtained by rotating the region in the first quadrant bounded by \( x = 4-y^2 \) and the y-axis about the y axis.
   e) The solid obtained by rotating the region in the first quadrant bounded by \( x = \sqrt{2}y(4-y^2) \) and the y-axis about the x axis.

11. \( \lim_{x \to 0^+} \frac{1}{x^3} \int_0^x \sin(t^2)dt \) is 
   a) 0  
   b) 1  
   c) \( \frac{1}{3} \)  
   d) 3  
   e) -3
12. A point on the curve \( y^2 = x^2 + 4x + 7 \) closest to the origin is
   a) \((-1, 2)\)
   b) \((1, \sqrt{12})\)
   c) \((0, \sqrt{7})\)
   d) \((2, \sqrt{19})\)
   e) \((-2, \sqrt{3})\)

13. Which of the following functions is differentiable at \( x = 0 \)?
   a) \( f(x) = \sqrt{1 + |x|} \)
   b) \( f(x) = |x| \)
   c) \( f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases} \)
   d) \( f(x) = \begin{cases} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \)
   e) \( f(x) = \begin{cases} \sin x & x \geq 0 \\ \cos x & x < 0 \end{cases} \)

14. If \( f(x) = x \ln x \) then \( f^{(10)}(x) \) is
   a) \( \frac{1}{10!} x^{10} \)
   b) \( 10! \ln(x) \)
   c) \( \frac{1}{8!} x^8 \)
   d) \( 8! \ln(x) \)
   e) \( 8! x^{-9} \)

15. The area between the curves \( y = 2x - x^2 \) and \( y = x^2 \) for \( 0 \leq x \leq 2 \) is
   a) \( -\frac{2}{3} \)
   b) \( -\frac{4}{3} \)
   c) 0
   d) 2
   e) \( \frac{2}{3} \)
Longer Questions

16. (50 pts) Evaluate the following definite integrals.

(i) \[ \int \frac{\sin 2x}{\sin x} \, dx \]

(ii) \[ \int \sqrt{3 - 5x} \, dx \]

(iii) \[ \int \frac{\sin^{-1} x}{\sqrt{1 - x^2}} \, dx \]

(iv) \[ \int x \sqrt{x^2 + a^2} \, dx \]

(v) \[ \int \frac{\cos \left( \frac{x}{\lambda} \right)}{x^2} \, dx \]
17. (60 pts) Evaluate the following definite integrals.

(i) \[ \int_0^3 |6 - 9x + 3x^2| \, dx \]

(ii) \[ \int_0^1 e^{xt} \, dt \]

(iii) \[ \int_0^1 \frac{1}{x^2 + 1} \, dx \]

(iv) \[ \int_e^4 \frac{dx}{x\sqrt{\ln x}} \]

(v) \[ \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{x^2 \sin x}{1 + x^6} \, dx \]
18. (50 pts) Evaluate the following limits.

(i) \( \lim_{n \to \infty} \frac{1}{n} \left( \sum_{i=1}^{n} \frac{i}{n^8} \right) \)

(ii) \( \lim_{n \to \infty} \frac{1}{n} \left( \sum_{i=1}^{n} \sqrt{1 - \frac{i^2}{n^2}} \right) \)

(iii) \( \lim_{x \to \infty} \frac{\ln x}{x^2} \)

(iv) \( \lim_{z \to 1^+} \frac{x - 1}{\tan \left( \frac{\pi x}{2} \right)} \)

(v) \( \lim_{x \to 0} \frac{\sin x}{1 + \cos x} \)
19. (50 pts) Evaluate the following derivatives.

(i) \( \frac{d}{dx} \left( \frac{x}{x^3 + 1} \right) \)

(ii) \( \frac{d^2}{dx^2} (e^{-x^2}) \)

(iii) \( \frac{d}{dx} \int_{x}^{e^x} \frac{\cosh(t^2)}{1 + t^4} \, dt \)

(iv) \( \frac{d}{dx} (x^{-x}) \)

(v) \( \frac{d^{30}}{dx^{30}} (\sin x) \)
20. (45 pts) The base of a solid is a square with vertices at (1,0), (0,1), (−1,0) and (0,−1). Each cross-section perpendicular to the x-axis is a semicircle. Find the volume of the solid.
21. (45 pts) Find the volume of the solid obtained by rotating the bounded region between the curves \( y = x^2 \) and \( y = x^3 \) about the line \( y = 2 \).