1. (30 points, 10 points apiece) Find the following. A correct answer will give full credit whether or not you show your computations. An incorrect answer, given with computations that are correct except for a minor error, will give partial credit.

(a) The locations of all critical points of the function \( x^2y - x - y \). (You are not asked to give the values of the function at these points.)

(b) \( \int_{x=1}^{2} \int_{y=0}^{1} \int_{z=y}^{x} xyz \, dx \, dy \, dz \).

(c) \( \int_{C} xy \, dx + \ln x \, dy \), where \( C \) is the curve \( x = e^t, y = e^{-t}, 0 \leq t \leq 1 \).

2. (16 points) Show that if \( f \) is a continuous function defined on \([0,1]\), then
\[ \int_{x=0}^{1} \int_{y=x}^{x+1} f(x) \, dy \, dx = \frac{1}{2} \int_{y=0}^{1} (y - y^2) \, f(y) \, dy. \]
(Suggestion: First change the order of integration.)

3. (18 points) (a) (6 points) Find real numbers \( a, b \) and \( c \) such that the vector field \( F(x, y) = \langle x^2y^5, ax^b \rangle \) on the plane is conservative.

(b) (6 points) Find a function \( f \) such that \( \nabla f = F \), where \( F \) is the vector field you found in part (a).

(c) (6 points) Find (by any method) \( \int_{C} F(x, y) \cdot dr \), where \( F \) is the vector field asked for in (a), and \( C \) is the path \( x = \sin t, y = 1 + \sin 2t, -\pi/2 \leq t \leq \pi/2 \).

4. (20 points) (a) (12 points) Express the integral \( \int_{x=1}^{2} \int_{y=x/2}^{x} (x/y^2) \sin(\pi x/y) \, dy \, dx \) as an integral in new variables \( u \) and \( v \), related to \( x \) and \( y \) by the equations \( x = u, y = u/v \). (You may take for granted that the above equations give a mapping that is one-to-one on its domain.)

(b) (8 points) Evaluate the integral of (a) by any method.

5. (16 points) Find the maximum and minimum values of the function \( x^2 + xy + y^2 \) on the circle \( x^2 + y^2 = 1 \), and list all points of the circle at which these values occur.