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Spring 1997, Math H1A
First Midterm

26 September, 1997
1:10-2:00 PM

1. (40 points, 8 points apiece) Compute each of the following. A correct answer gives full credit whether or not you show your computations. An incorrect answer, given with computations that are correct except for a minor error, will get partial credit.

(a) $\lim_{x \to 2} (x^3 - 8)/(x^2 - 4)$. (Give a real number, or $+\infty$, or $-\infty$, or say "Undefined & not $+\infty$ or $-\infty$".)

(b) $\lim_{x \to -\infty} (x^3 - 8)/(x^2 - 4)$. (Same choices as for (a).)

(c) $\frac{d}{dx} x^5 \sin x$.

(d) $\frac{d^2}{dx^2} f(1/x)$, where f is twice differentiable.

(e) The equation of the line tangent to the curve $x^2 - y^2 = 5$ at the point $(3, 2)$.

2. (10 points) Complete the following *precise* definition: *Let* f *be a function defined on the real line, and* L *a real number. Then* $\lim_{x \to -\infty} f(x) = L$ *means that*

3. (20 points) For $F(x) = f(x)g(x)$, derive using the definition of derivative the formula $F'(a) = f(a)g'(a) + f'(a)g(a)$, where f and g are differentiable at a. You may use facts we have proved about limits, but not further facts proved about derivatives (such as the above formula).

4. (30 points) Suppose p and q are polynomials, and n a positive integer. Prove that $\frac{d^n}{dx^n} p(x)/q(x)$ can be written with denominator $q(x)^{n+1}$; i.e., that there is a polynomial $a(x)$ such that $\frac{d^n}{dx^n} p(x)/q(x) = a(x)/q(x)^{n+1}$.

You may take for granted that a sum, product, or difference of polynomials is a polynomial, and that the derivative of a polynomial is a polynomial, and thus that any expression obtained from polynomials by any combination of these operations is a polynomial. You may also assume any results proved in the course so far. Suggestion: Use mathematical induction. (You will get partial credit for merely setting up the induction, i.e., for writing down what has to be proved to give such a proof. Once you write this down, the result will be a fairly straightforward computation. If you can't give a full proof, you can get some further partial credit for verifying the result for $n = 1, 2, 3$.)