1. (45 points, 5 points apiece) Give each of the following if it is defined. If an expression is undefined, say so. (You do not have to give a reason in such cases.)

(a) An equation for the tangent plane to the surface \( x^2 - 2y^2 + 3z^2 = 20 \) at the point (1, 2, 3).

(b) An expression for \( \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} f(x, y) \, dy \, dx \) in which the order of integrations is reversed (i.e., as an iterated integral of \( f(x, y) \) in which the outer integration is with respect to \( y \) and the inner integration with respect to \( x \)).

(c) The mass of a lamina occupying the quarter-circular region \( \{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 1 \} \), and having density function \( \rho(x, y) = xy \).

(d) A sketch of the curve \( x = \sin 2t, \ y = \sin t, \ 0 \leq t \leq 2\pi \), with the maximum and minimum values of \( x \) and \( y \) marked on the axes.

(e) The unit upward normal to the surface \( x = u \cos v, \ y = u \sin v, \ z = v \) at the point \( u = 3/4, \ v = 0 \).

(f) \( \nabla F \), where \( F(x, y, z) = <1, x, xy> \).

(g) \( \text{div} F \), where \( F(x, y, z) = <x^3 y^5, x^8, y^{13}, x^2 y^{34}> \).

(h) An equation of the form \( z = f(x, y) \) describing the set of points \( (x, y, z) \) equidistant from the line \( x = z = 0 \) (the y-axis) and the line \( y = 0, z = 1 \).

(i) \( \int_S F \cdot dS \), where \( F \) is the constant vector field \( <2, 5, -1> \), and \( S \) is the parallelogram with vertices \( (0, 0, 0), (0, 2, 2), (1, 2, 2), (1, 0, 0) \), and downward orientation.

2. (14 points) Let \( C \) be a space curve, given by

\[ r(t) = <x(t), y(t), z(t)> \quad (a \leq t \leq b), \]

where \( x(t), y(t), \) and \( z(t) \) are functions; and let \( S \) be the surface given parametrically by

\[ q(s, t) = sr(t) \quad (0 \leq s \leq 1, \ a \leq t \leq b). \]

(This will be a "cone-like" surface, with the origin as apex and the curve \( C \) as "base".)

Assuming that this surface does not "overlap" itself, i.e., that the above parameterization is one-to-one for \( s \neq 0 \), show that the area of \( S \) is given by

\[ A = \int_a^b \sqrt{(xy' - x'y)^2 + (yz' - y'z)^2 + (zx' - z'x)^2} \, dt, \]

where \( x \) means \( x(t) \), \( y' \) means \( y'(t) \), etc.
3. (13 points) Let \( \mathbf{F} \) be a differentiable vector field on \( \mathbb{R}^3 \), let \( f \) be the function 
\[
f(x, y, z) = 1 - x^2 - y^2 - z^2,
\]
and let \( E \) be the solid ball 
\[
E = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}.
\]
Show that 
\[
\iiint_E \text{div}(f(x, y, z) \mathbf{F}(x, y, z)) \, dx \, dy \, dz = 0.
\]

4. (14 points) Suppose \( f(x, y) \) is a real-valued function of two real variables \( x \) and \( y \).

(a) (4 points) Given a point \( (x_0, y_0) \), and a real number \( L \), define what it means for 
\[
\lim_{(x, y) \to (x_0, y_0)} f(x, y) = L
\]
to hold.

(b) (3 points) Define what it means for \( f \) to be continuous at \( (x_0, y_0) \).

(c) (7 points) Let \( f \) be the function defined by the formulas 
\[
f(x, y) = x \text{ if } x^2 + y^2 \leq 1,
f(x, y) = y \text{ if } x^2 + y^2 > 1.
\]
At what points is \( f \) continuous, and at what points is it discontinuous? The answer will involve more than one case; give a reason for one case of your answer.

5. (14 points) Let \( \mathbf{F} \) be the vector field \( \langle x^2e^z, 0, -2xe^z \rangle \).

(a) (5 points) Find constants \( a \) and \( b \) such that \( \text{curl} \langle axye^z, 0, bx^2ye^z \rangle = \mathbf{F} \).

(b) (9 points) Calculate \( \iint_S \mathbf{F} \cdot d\mathbf{S} \), where \( S \) is the surface \( z = xy(1 - x - y) \), \( x \geq 0, y \geq 0, x + y \leq 1 \), with upward orientation. (Suggestion: Use part (a).)