Math 113: Introduction to abstract algebra.
H. W. Lenstra, Jr. (879 Evans, tel. 643-7857, e-mail hwl@math).
Fall 1995
Make-up midterm, Monday, December 11, 12:30–3:30 p.m., 60 Evans.

Name:

Note. You have to do three out of the four problems. Cross out the problem that you don’t want to be graded. Give complete proofs of the assertions you are making and of the correctness of your answers. Theorems proved in the book or in class may be used without proof (but do give the formulation).

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Problem 1.
What is the smallest $n$ for which there exists an element of order 12 in the alternating group $A_n$?

Solution:
Problem 2.
Let $G$ be the group $\mathbb{Z} \times \mathbb{Z}_5$, and let $a = (1, 1) \in G$. What is the order of the group $G/\langle a \rangle$?

Solution:
Problem 3.
Denote by $\mathbb{C}^*$ the multiplicative group of non-zero complex numbers and define the homomorphism $f: \mathbb{C}^* \to \mathbb{C}^*$ by $f(z) = z/\bar{z}$, where $\bar{z}$ denotes the complex conjugate of $z$. (You do not have to prove that $f$ is a homomorphism.)

Determine the image of $f$ and the kernel of $f$, and prove that

$$\mathbb{C}^*/\mathbb{R}^* \cong T,$$

where $\mathbb{R}^*$ is the multiplicative group of non-zero real numbers and $T$ is the circle group: $T = \{z \in \mathbb{C} | |z| = 1\}$.

Solution:
Problem 4.
How many homomorphisms are there from the permutation group $S_3$ to the Klein four group $V_4$?

Solution: