This is a closed book exam. You are allowed one 2-sided 8½" × 11" sheet of notes. Attempt all problems. Write solutions on these sheets. Ask for scratch paper if the fronts and backs of these pages are not sufficient; put your name on any such extra sheets and hand them in with your exam.

Credit for an answer may be reduced if a large amount of irrelevant or incoherent material is included along with the correct answer.

Questions begin on the next sheet. Fill in your name and section on this sheet now, but do not turn the page until the signal is given. At the end of the exam, stop writing and close your exam as soon as the ending signal is given, or you will lose points.

Think clearly, stay calm.

Your name

Sections: Mark yours with ×.
(in order of hour, not section-number.)

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| 171 Stanley 8:00-9:00 201 | Benjamin Tsou
| 3102 Etcheverry 9:00-10:00 203 | Kiril Datchev
| 71 Evans 10:00-11:00 204 | Benjamin Tsou
| 3111 Etcheverry 11:00-12:00 205 | Harold Williams
| 75 Evans 12:00-1:00 206 | Koushik Pal
| 70 Evans 1:00-2:00 207 | Gary Sivek
| 105 Latimer 2:00-3:00 208 | Gary Sivek
| 3102 Etcheverry 2:00-3:00 211 | Koushik Pal
| 85 Evans 5:00-6:00 210 | Harold Williams
| Other or none | Explain

Leave blank for grading

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MUTTS Patrick McDonnell
1. (24 points: 4 points each.) Compute the following.

(a) \[ \int \frac{dx}{x^3 + x} \].

(b) \[ \sum_{n=0}^{\infty} \frac{2^n + 3^n}{n!} \].

(c) The set of real numbers \( x \) such that \( \sum_{n=0}^{\infty} n^5 x^n \) converges.
(d) The general solution to the differential equation
\[ y' = x^2 y^3. \]

(e) The general solution to the differential equation
\[ y'' + 4y' - 21y = 0. \]

(f) The general solution to the differential equation
\[ 2y'' + y = x^2. \]
2. (28 points: 7 points each.) Compute the following.

(a) An arc-length function for the curve \( y = e^x \) (i.e., a function \( F(x) \) such that for \( a < b \), the length of the curve from \( x = a \) to \( x = b \) is given by \( F(b) - F(a) \)).

(b) The centroid of the region bounded above by the curve \( y = x^n \), below by the \( x \)-axis, and on the right by the line \( x = 1 \); where \( n \) is a positive integer.
(2, continued) 

work

answers:

(c) The solution to the differential equation 
\[ y' = y \sin x + \sin x \] satisfying \( y(0) = 0. \)

(d) The general solution to the differential equation 
\[ y'' - 2y' - 3y = e^{3x} + 10 \sin x. \]
4. (14 points) Prove that if $a_0, a_1, a_2, \ldots, a_n, \ldots$ is a bounded sequence of real numbers, then the series $\sum_{n=0}^{\infty} \frac{a_n}{2^n}$ is absolutely convergent.

In proving this, you may call on any facts about series in our text.
5. (13 points) Recall that if $f$ is a function with $n+1$ derivatives on an interval $I$, and $a$ is a point of $I$, then the $n$-th degree Taylor approximation of $f(x)$ about $a$ means the polynomial

$$T_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!} (x-a)^i,$$

and Taylor's formula says that for every point $x$ of $I$, the remainder $R_n(x) = f(x) - T_n(x)$ equals $f^{(n+1)}(z)(x-a)^{n+1}/(n+1)!$ for some $z$ between $a$ and $x$.

(I have slightly simplified the statement in the text, removing the assumption that $x$ is distinct from $a$, and hence the statement that $z$ lies strictly between $a$ and $x$. It remains true in this simplified form.)

Write out the second degree Taylor approximation for $e^x$ about $a = 2$, and verify that for all $x \in [1.5, 2.5]$, Taylor's formula shows that this polynomial approximates $e^x$ to within $\leq e^{5/2}/48$.

$$T_2(x) = \ldots$$

Verification of error bound: