Math 104: Midterm 2

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Name :

Student ID Number:

Instructions: This is a closed-book test. Each problem is worth 20 points. Read the questions carefully, and show all your work. All work should be done on the exam paper. Additional white paper is available if needed. Good luck.

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

(1) State whether these series converge or diverge and explain why.

(a) $\sum_{n=0}^{\infty} \frac{1-n}{1+2n}$

(b) $\sum_{n=0}^{\infty} \frac{n^4}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{x^n}{n^n}$, for any real x.

(2) (a) Define **absolute convergence** of a function f.

(b) If $\sum b_n$ converges absolutely and if $\lim_{n\to\infty} \frac{|a_n|}{|b_n|}$ exists, show that $\sum a_n$ converges absolutely.

(3) Let f be a real function, defined as $f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$. Prove that f is not continuous at any point.

(4) Assume f is a real-valued function on $\mathbb{R}$ and $\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0$. Show that f is uniformly continuous.

(5) (a) Define **the derivative** of a function f at a point x.
(b) Let f and g be functions which are differentiable at x. State and prove the product rule for the derivative of fg at x.

(6) (a) Let f be a continuous real valued function on $[a, b]$. Assume $f(a) = f(b) = 0$, and that $f'(x)$ exists for all $x \in [a, b]$. Show that there exists a $c \in (a, b)$, where $f'(c) = 0$. (This is called Rolle's Theorem. Prove the statement directly; don't just claim that it is a corollary of another theorem.)
(b) Does $g(x) = 1 - |x|$ on $[-1, 1]$ satisfy the above theorem? Explain.

(7) (a) State two equivalent definitions for the number e. (One as a series and the other as a limit of a sequence.)

(b) Let E be a bounded, noncompact set in $\mathbb{R}$. Find a continuous function of E which is not bounded and which is not uniformly continuous. Explain your answer.