Math 1A Midterm 1 2005-9-20 11:00-12:30pm. R. Borchers

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for a clear and correct answer.

1. Find the domain of the function \( g(u) = \sqrt{u + \sqrt{2 - u}} \).

2. Sketch the graph of \( y = |x^2 - 2x| \).

3. Find a formula for the inverse of the function \( f(x) = 1 + e^{x^3} \).

4. Sketch the graph of a function \( f \) that satisfies the conditions
   \[ \lim_{x \to 0^-} f(x) = 1, \lim_{x \to 0^+} f(x) = -1, \lim_{x \to -1^-} f(x) = 1, \lim_{x \to -1^+} f(x) = -1, f(2) = 1. \]

5. Evaluate the limit
   \[ \lim_{x \to -1} \frac{x^3 - 1}{x^2 - 1} \]

6. How close to 2 do we have to take \( x \) so that \( 5x + 3 \) is within a distance of 0.01 from 13?

7. Find the numbers at which \( f \) is discontinuous, where \( f \) is defined by \( f(x) = x^2 \) if \( x \leq 1 \), \( f(x) = 1/x \) if \( 1 < x < 3 \), \( f(x) = 1/2 + \sqrt{x - 3} \) if \( x \geq 3 \).

8. What is
   \[ \lim_{x \to -\infty} \frac{(3x + 1)(4x + 1)}{(x + 1)(2x + 1)} \]

9. A curve has equation \( y = f(x) \). Write an expression for the slope of the secant line through the points \((3, f(3))\) and \((x, f(x))\), and write an expression for the slope of the tangent line at \((3, f(3))\).

10. If \( g(x) = x^3 + x^2 + x + 1 \), find \( g'(0) \) and use it to find an equation of the tangent line to the curve \( y = x^3 + x^2 + x + 1 \) at the point \((0, 1)\).

11. Sketch the graph of the function \( y = x^2|x| \), say where it is differentiable, and find a formula for its derivative.

12. Differentiate the function \( y = e^{x+1} + 4x^2 + (x + 1)/\sqrt{x} \)

13. At what point on the curve \( y = 1 + 2e^x - 3x \) is the tangent line parallel to the line \( 3x - y = 5 \)?

14. Differentiate \( x e^{x} (\sqrt{x} + 1) \)

15. Differentiate
   \[ \frac{x^2}{1 + 1/x} \]