Math H53 Midterm Exam 2

November 5th, 2003

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1 a: (5 pts) Use Lagrange multipliers to find the maximum of \( f(x, y) = x^2 - y^2 \)
given the constraint \( x^2 + 2y^2 = 1 \).

2 a: (5 pts) Evaluate
\[
\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dx \, dy
\]
by converting to polar coordinates.

b: (5 pts) Let \( f(x, y) = |x| + |y| \). Find the Fenchel subdifferential \( \partial_c(f(x, y)) \).

3 a: (5 pts) Let \( \mathcal{A} \) be the set of all possible stories written in English. Show that
\( \mathcal{A} \) is countable.

Bonus: (2 pts) Show the set of all possible illustrated stories is uncountable.

4 a: (5 pts) Let \( \langle u(x, y), v(x, y) \rangle \) be a conservative vector field, with a potential
function \( U(x, y) \). Let \( \langle v(x, y), u(x, y) \rangle \) be a conservative vector field, with a poten-
tial function \( V(x, y) \). Show that \( \langle U(x, y), V(x, y) \rangle \) and \( \langle V(x, y), U(x, y) \rangle \) are both
conservative vector fields.

b: (5 pts) Reverse the order of integration of
\[
\int_0^1 \int_{x^2}^1 f(x, y) \, dx \, dy.
\]
(Do not evaluate.)

5: (5 pts) Let
\[
f(x, y) = \frac{2}{3} (x-1)^{\frac{3}{2}} + \frac{2}{3} y^{\frac{3}{2}}.
\]
Find the surface area of \( f(x, y) \) over the region \( D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1 \} \).

6: (5 pts) Let \( T : \mathbb{R} \to \mathbb{R}^2 \) by \( T(x, y) = (e^x \cos y, e^x \sin y) \) Let \( R = \{(x, y) : 1 \leq x^2 + y^2 \leq 4 \} \). Us the change of variables \( T \) to evaluate
\[
\iint_R \frac{1}{\sqrt{u^2 + v^2}} \, du \, dv
\]