Math 128a Midterm Exam 2
K. Hare
October 24, 2003

NAME (printed) : 

(Last Name) (First Name) 

Signature : 

Student Number : 

(1) Do NOT open this test booklet until told to do so
(2) Do ALL your work in this test booklet
(3) Show ALL your work
(4) Check that there are 6 problems and 7 pages (including this one)
(5) NO CALCULATORS
(6) Please keep your arms and legs inside the ride at all times.

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1 a: (4 pts) Consider

\[ A = \begin{bmatrix} 1 & 4.25 & 1.25 \\ 4 & 1 & 1 \\ 1 & 1.25 & 4.5 \end{bmatrix} \]

Use Gaussian elimination, with partial pivoting to compute the determinant of \( A \).

b: (3 pts) If it takes 10 seconds to compute the determinant of a random 1000 \( \times \) 1000 matrix, how long would it take to compute the determinant of a random 5000 \( \times \) 5000 matrix?
2 a: (4 pts) A matrix $A$ is positive definite if $x^tAx > 0$ for all $x \neq 0$. Prove that the diagonal entries $a_{i,i} > 0$

b: (3 pts) Find $\alpha > 0$ such that the following system is strictly diagonally dominate.

$$
\begin{bmatrix}
\alpha & 2 & 3 \\
10 & 20 & 7 \\
\alpha & 3 & 10
\end{bmatrix}
$$
3 a: (4 pts) Let $A$ be a $n \times n$, non-singular lower triangular matrix. How many step of the Jacobi Iterative method are needed to solve $Ax = b$? (Justify your answer.)

b: (4 pts) Compute the first two steps of the Jacobi Iterative method, with starting point $(0,0)$, to the system

$$\begin{bmatrix} 10 & 3 \\ 2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$
4 a: (4 pts) Given \( f(-1) = -4 \), \( f(0) = -3 \) and \( f(1) = 0 \), use Neville’s method to approximate \( f(2) \).

b: (5 pts) Use a variation of the Newton Divide Difference method for Hermite polynomials to find the unique polynomial, of degree at most three, such that

\[
P(-1) = -4, \ P(0) = -1, \ P'(0) = 2, \ P(1) = 2
\]
5: (5 pts) A natural cubic spline \( S \) on \([0, 2]\) is defined by

\[
S(x) = \begin{cases} 
  x^3 & \text{if } 0 \leq x \leq 1 \\
  a + b(x - 1) + c(x - 1)^2 + d(x - 1)^3 & \text{if } 1 \leq x \leq 2
\end{cases}
\]

Find \( a, b, c \) and \( d \).
6 a: (4 pts) Complete the factorization below
\[
\begin{bmatrix}
2 & 0 & -1 \\
4 & -3 & -5 \\
-2 & 0 & 3 \\
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
\_ & \_ & 0 \\
\_ & \_ & 1 \\
\end{bmatrix}
\begin{bmatrix}
\_ & \_ & \_ \\
0 & 1 & \_ \\
0 & 0 & \_ \\
\end{bmatrix}
\]

b: (4 pts) Prove that there do not exist lower and upper triangular matrices \( L \) and \( U \) satisfying
\[
\begin{bmatrix}
0 & -2 & 0 \\
2 & 1 & 0 \\
6 & 2 & -1 \\
\end{bmatrix}
\]