1. Let the sequence \((x_n)\) be defined by:
   \[ x_1 = 1, \quad x_{n+1} = 1 + x_n^2. \]
   Prove that \((x_n)\) is monotone and find its limit.

2. Let the sequence \((x_n)\) be defined by:
   \[ x_n = \frac{a^n}{a^n + b^n} \]
   where \(a, b > 0\) are fixed numbers.
   Prove that \((x_n)\) is convergent and find its limit.

3. Let the sequence \((a_n)\) be defined by:
   \[ a_n = \frac{1}{1+2+\cdots+n}. \]
   a) Prove by induction that \(a_n = \frac{2}{n(n+1)}\), \(\forall n \geq 1\).
   b) Prove that the series \(\sum_{n=1}^{\infty} a_n\) is convergent.
   c) Find the sum of the series at 3).

4. Let \(f: \mathbb{R} \to \mathbb{Z}\) be a continuous function.
   Prove that \(f\) is constant i.e. \(\exists c \in \mathbb{R}\) such that \(f(x) = c\), \(\forall x \in \mathbb{R}\).