Math 53 Midterm #1, 10/2/03, 8:10 AM – 9:30 AM Hutchings

No calculators or notes are permitted. Each of the 6 questions is worth 10 points. Please write your solution to each of the 6 questions on a separate sheet of paper with your name on it. Please put a box around the final answer and, to maximize credit, show your work. Good luck!

1. (a) Sketch the curve given in polar coordinates by the formula
   \[ r = e^{2\theta}, \quad 0 \leq \theta \leq 2\pi. \]

   (b) Compute the length of the above curve.

2. Show that the limit
   \[ \lim_{(x,y) \to (1,1)} \frac{x + 2y - 3}{x + y - 2} \]
   does not exist.

3. Suppose the function \( f(x, y) \) satisfies
   \[ \frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial f}{\partial y} = \frac{1}{y}. \]
   Suppose as usual that \( x = r \cos \theta \) and \( y = r \sin \theta \). Calculate \( \partial f/\partial r \) and \( \partial f/\partial \theta \) in terms of \( r \) and \( \theta \).

4. (a) Sketch the surface \( x^2 + (y - 1)^2 = z^2 \).

   (b) Find the tangent plane to the above surface at the point (4, 4, 5).
      (Write your answer in the form \( ax + by + cz = d \).)

5. (a) Find normal vectors to the planes
   \[ 2x + 2y + z = 5, \quad 2x - y - 2z = -1. \]

   (b) The above two planes intersect along a line \( L \). At what angle do the two planes intersect? \textit{Hint:} this is the angle between the above two normal vectors.

   (c) Find a tangent vector to the above line \( L \). \textit{Hint:} this must be perpendicular to the above two normal vectors.

6. Suppose \( z \) is implicitly defined as a function of \( x \) and \( y \) by the formula
   \[ z e^{x+2y+3z} = 4. \]
   Calculate \( \partial z/\partial x \) and \( \partial z/\partial y \) in terms of \( x, y, \) and/or \( z \).