Math H53 Midterm Exam 1

October 3rd, 2003

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1 a: (5 pts) Let \( f(t) = (3t^2, 2t^3) \). Find the length of \( f(t) \) between \( 0 \leq t \leq 2 \).

b: (5 pts) Find all solutions to \( z^4 = -4 \).

2: (5 pts) Let \( f \) be a harmonic function with continuous partial derivatives of any order. Further let \( f_x(x, y) = 2x + y \). Find \( f_y(x, y) \).

3: (5 pts) Let \( r(\theta) = 4 \sin(3\theta) \). Find the area of the curve in one loop.

4 a: (5 pts) Consider the function \( f(x, y) = x^4 + y^4 + x^2y^2 - xy + 3 \). Find the direction of steepest descent from the point \( (1, 2) \)

b: (5 pts) Given
\[
2x + 4x + 4y^2 - 24y = x^2 + 2x
\]
Convert this to standard form. What sort of quadratic surface is this. (If you can’t remember the name, just draw a picture)

5: (5 pts) Let \( V \) be the vector space of polynomials. Define the dot product (inner product) between two vectors \( f \) and \( g \) as
\[
\int_0^1 f(x)g(x)dx
\]
Find \( g(x) \) orthogonal to \( f(x) = x \).

6: (5 pts) Let \( f : \mathbb{R}^2 \to \mathbb{R} \) be a continuous functions, with continuous partial derivatives. Let \( \{(x_i, y_i)\}_{i=0}^{\infty} \) be a sequence of local maximums. Further let \( \lim_{i\to\infty} x_i = c \) and \( \lim_{i\to\infty} y_i = d \). Show that \( f \) has a critical point at \( (c, d) \).

Bonus: (2 pts) Give an example to show that \( (c, d) \) can be a local minimum.