Math 110, Section 7 October 10, 2003
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1st MIDTERM EXAMINATION

Open book, open notes. In your proofs on the exam, you may use any result from Chapters 1–4 of our textbook or from the lectures. Be sure to state clearly any result you use.

The points for each problem are in parentheses.

1. (10) Is the set $U = \{p \in \mathcal{P}(\mathbb{F}) : p(0) = p'(1)\}$ a subspace of $\mathcal{P}(\mathbb{F})$? Justify your answer.

2. (10) Let v_1, v_2, v_3, v_4 be linearly independent vectors in a vector space V. Prove that the vectors v_1, $2v_1 + v_2$, $3v_1 + 2v_2 + v_3$, $4v_1 + 3v_2 + 2v_3 + v_4$ are linearly independent.

3. (10) Let the vector space V be finite dimensional, and let T be in $\mathcal{L}(V)$. Prove that null $T+$ range $T = V$ if and only if (null T) $\cap$ (range T) $= \{0\}$.

4. (20) Consider the four-dimensional vector space $\mathcal{P}_3(\mathbb{F})$, consisting of the polynomials with coefficients in $\mathbb{F}$ whose degrees are at most 3. Let p_0, p_1, p_2, p_3 be the standard basis for $\mathcal{P}_3(\mathbb{F}) : p_k(z) = z^k$ $(k = 0, 1, 2, 3)$. Let the linear transformation T in $\mathcal{L}(\mathcal{P}_3(\mathbb{F}))$ be defined by $(Tp)(z) = p(z+1)$.

    (a) Find the matrix for T with respect to the standard basis.

    (b) Prove that T is invertible.

    (c) Find the matrix for T^{-1} with respect to the standard basis.