1. Let \( a, b \in \mathbb{R} \) such that \( ab > 0 \) and consider \( f : [a, b] \rightarrow [a, b] \) a continuous function. Prove that there exists \( c \in (a, b) \) such that \( c \cdot f(c) = a \cdot b \).

2. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \), \( f(x) = x \cdot (\sin(x^2))^2 \). Prove that \( f \) is continuous, but it is not uniformly continuous.

3. Consider the sequence of functions \( f_n : \mathbb{R} \rightarrow \mathbb{R} \), \( f_n(x) = \frac{x}{1 + nx^2} \). Prove that \( (f_n) \) converges pointwise to \( f : \mathbb{R} \rightarrow \mathbb{R} \), \( f(x) = 0 \), \( \forall x \in \mathbb{R} \). Is this convergence also uniform?

4. Consider the power series \( \sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1} \). Find the radius of convergence and the sum of this series.

5. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) a function differentiable at \( 0 \). Define \( g : \mathbb{R} \rightarrow \mathbb{R} \) by \( g(x) = f(|x|) \). Prove that \( g \) is differentiable at \( 0 \) if and only if \( f'(0) = 0 \).

6. Consider \( f : \mathbb{R} \rightarrow \mathbb{R} \), \( f(x) = \begin{cases} 2 \cos x - \sin x & , x > 0 \\ ax^2 + bx + c & , x \leq 0 \end{cases} \). Find \( a, b, c \) such that \( f \) is two times differentiable.

7. Let \( f : [0, 1] \rightarrow \mathbb{R} \) continuous function such that \( \int_{0}^{1} f(x) \, dx = \frac{1}{3} \). Prove that \( \exists c \in (0, 1) \) such that \( f(c) = c^2 \).