1. (20%) A rhombus is a quadrilateral with four congruent sides. Prove that the diagonals of a rhombus are perpendicular to each other.

2. (25%) Assume you know that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and is half the length of the third side. Prove that if $BB'$ and $CC'$ are medians of triangle $ABC$, then their point of intersection $G$ satisfies $|BG| = 2|GB'|$, $|CG| = 2|GC'|$.

3. (30%) Assume you know that an isometry of $\mathbb{R}^2$ with three distinct fixed points is necessarily the identity map. Prove that every isometry of $\mathbb{R}^2$ is a bijection.

4. (25%) Given $\angle AOB$, where $O$ is the origin of $\mathbb{R}^2$. Let $\alpha$, $\beta$ be positive numbers and let $P = \alpha A + \beta B$. Prove that $P$ is in the interior of $\angle AOB$. Is the positivity of $\alpha$ and $\beta$ really necessary for this conclusion to hold?