1. (48 points; 12 points each.) For each of the items listed below, either give an example, or give a brief reason why no example exists. (If you give an example, you do not have to prove that it has the property stated. Examples should be specific for full credit; i.e., even if there are many objects of a given sort, you should name one.)

(a) A group $G$, and two nonisomorphic finite $G$-sets with the same number of elements.

(b) A surjective group homomorphism from the alternating group $A_6$ to the alternating group $A_5$.

(c) Groups $G$ and $H$ and a homomorphism $f : G \to H$ which is one-to-one, but such that there does not exist a homomorphism $g : H \to G$ for which $gf$ is the identity endomorphism of $G$.

(d) A finite group $G$, a prime $p$, and two distinct $p$-Sylow subgroups of $G$.

2. (18 points) Prove: If $p$ and $q$ are primes such that $q \equiv 1 \pmod{p}$, then there exists a noncommutative group of order $pq$. (This was proved in the Companion. In proving it here, you may assume any results proved in Lang, and any proved in the Companion up to the point where that result was obtained.)

3. (16 points) Let $I$ and $J$ be ideals of a commutative ring $A$. Show that if the ideal $I \cap J$ is prime, then either $I \subseteq J$ or $J \subseteq I$.

4. (18 points) Show that if $f : G \to F$ is a surjective group homomorphism, where $F$ is a free group and $G$ an arbitrary group, then there exists a group homomorphism $g : F \to G$ such that $fg$ is the identity endomorphism of $F$. 