Midterm #2
Math 121A (Section 2) - Fall 2001  M. Tokman

Each problem counts 20 points

**Problem # 1.** Given the equations

\[ s + t = x, \]
\[ s^2 + t^2 = y \]

compute \( \left( \frac{dx}{d\theta} \right)_\theta \) at the point \( (s, t, x, y) = (1, -1, 0, 2). \)

**Problem # 2.** Let

\[ f(z) = \frac{1}{(z + 2)(z + 1)} + e^z, \]

(a) Identify all the singularities of \( f(z) \) and specify the type of each of the singularities.

(b) Compute all possible Laurent series expansions of \( f(z) \) around \( z = 0 \) and specify the region of convergence for each of the series. \( \text{(Hint: The series } (1 + w)^p = 1 + pw + \frac{p(p-1)}{2!}w^2 + \frac{p(p-1)(p-2)}{3!}w^3 + \ldots, \text{where } p = -1, -2, \ldots, \text{converges for } |w| < 1). \)

(c) Evaluate residues of \( f(z) \) at \( z = 0, z = -1 \) and \( z = 3. \)

**Problem # 3.** Evaluate the following integral using the residue theorem:

\[ \int_{0}^{\infty} \frac{x^{1/3}}{x^2 + 1} \, dx. \]

**Problem # 4.** (a) Define what it means for a function \( f(z) = u + iv \) to be analytic at a point \( z = z_0. \)

(b) Assume \( f(z) \) is analytic at \( z = z_0. \) State and derive the **Cauchy-Riemann conditions**.

(c) Show that if the function \( f(z) = u + iv \) is analytic in some region of a complex plane then its real and imaginary parts are harmonic functions in that region.

**Problem # 5.** The temperature \( T \) at each point \( (x, y) \) of a circular plate \( x^2 + y^2 \leq 1 \) is given by \( T = 2x^2 - 3y^2. \) Find the hottest and coldest points of the plate.