Professor Borcherds 1st Midterm
Math 121 Midterm, Thursday September 27, 9:40-11:00.

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 page of notes.
Answer as many questions as you can.
All questions have about the same number of marks.

1. Test the series $\sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)}$, $\sum_{n=2}^{\infty} \frac{1}{n \log(n)}$ and $\sum_{n=1}^{\infty} \frac{100^n}{n!}$ for convergence.

2. Evaluate $\lim_{x \to 0} \frac{1 - e^{x^3}}{x^3}$.

3. Express each of the following complex numbers in the form $a + ib$ for $a$ and $b$ real: $(1 + i)/(2 + i)$, $(1 + i)^{10}$, $e^{3\pi i/2}$.

4. Evaluate $1 + \cos(\theta) + \cos(2\theta) + \cdots + \cos(n\theta)$.

5. If $x^3 + ay = b$ and $y^3 + bx = a$ find $(\partial x/\partial a)_b$, $(\partial x/\partial a)_y$, $(\partial a/\partial x)_b$, $(\partial a/\partial x)_y$ at $(x, y, a, b) = (-1, 2, 3, 5)$.

6. If $xe^y = ye^x$ find $dy/dx$ and $d^2y/dx^2$ for $y \neq 1$.

7. Find all possible values of $i^{1+i}$.

8. Find a formula for $\cos(5\theta)$ as a polynomial in $\cos(\theta)$ and $\sin(\theta)$. 