Midterm #1
Math 121A (Section 2) - Fall 2001  Tekman

Each problem counts 10 points

Problem # 1. Find all solutions of the equation

\[ z^3 = -8i, \]

write them in a polar \(|z|e^{i\text{arg}(z)}\) and rectangular \((\text{Re}(z) + i\text{Im}(z))\) forms and plot them in the complex plane.

Problem # 2. Find the sum of the series

\[ \left(\pi/6 + i\right)^3 + \left(\pi/6 + i\right)^5 + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\pi/6 + i\right)^{2n+1}}{(2n+1)!} \]

and write it in a rectangular form \(\text{Re}(z) + i\text{Im}(z)\).

Problem # 3. Find the first three terms of the two-variable Maclaurin series for

\[ \frac{\sinh(xy)}{1 + x^2y}. \]

Write down both the derivation and the final answer!

Problem # 4. (a) Derive the formula

\[ \tanh^{-1}(z) = \frac{1}{2} \ln \frac{1 + z}{1 - z} \]

from the definition of \(\tanh z\) (or definitions of \(\sinh z\) and \(\cosh z\)).

(b) Use the derived formula to compute

\[ (i)^2 \tanh^{-1}(i). \]
Problem # 5. Find the interval of convergence of the following series (including end points tests!):

\[ \sum_{n=1}^{\infty} \frac{(x + 2)^n}{\sqrt{n}(-3)^n}. \]

Justify your answer by mentioning the theorems/tests that you use to draw conclusions about convergence, and state explicitly if the convergence is absolute or conditional.

Problem # 6. Compute \( dz/dt \) given

\[
\begin{align*}
z &= x^y, \\
x &= \sin t, \\
y &= \tan t.
\end{align*}
\]

Problem # 7. Does the following series converge?

\[ \sum_{n=1}^{\infty} \frac{n^3 - \ln n}{2^n + 10n} \]

Justify your answer by mentioning the theorems/tests that you use to draw conclusions about convergence.