1 (3 points). Calculate the number of primitive roots mod $35035 = 5 \cdot 7^2 \cdot 11 \cdot 13$.

2 (6 points). What is the remainder when one divides the prime number $1234567891$ by $11$? What is the remainder when $11^{1234567890}$ is divided by $1234567891$?

3 (5 points). Find a mod 29 inverse to the $2 \times 2$ matrix \[
\begin{pmatrix}
1 & 2 \\
3 & 9
\end{pmatrix}
\] mod 29.

4 (9 points). If $p$ is a prime, show that all prime divisors of $2^p - 1$ are congruent to $1$ mod $p$. (For example, $2^{11} - 1 = 23 \cdot 89$ is divisible by the primes 23 and 89 and by no others.)

5 (7 points). Let $\mu$ be the Möbius function, and let $\tau$ be the function whose value on $n \geq 1$ is the number of divisors of $n$. Explain why the function $F(n) := \sum_{d|n} \mu(d)\tau(d)$ satisfies the relation $F(n_1n_2) = F(n_1)F(n_2)$ when $\gcd(n_1, n_2) = 1$. Calculate $F(p^e)$ when $p$ is a prime and $e$ is a positive integer.