April 17, 2003

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Learning about reality from observation

2400 years ago Plato asked what we can learn from seeing only shadowy images of reality. In the 1930's Whitney studied "typical" images of manifolds in $\mathbf{R}^m$ and asked when the image was homeomorphic to the original. Let A be a closed set in $\mathbf{R}^n$ and let f:$\mathbf{R}^n$---->$\mathbf{R}^m$ be a "typical" smooth map where $n>m$. (Plato considered only the case $n=3$, $m=2$.) Whitney's question has natural extensions. If $f(A)$ is a bounded set, can we conclude the same about A? When can we conclude the two sets have the same cardinality or the same dimension (for typical f)? (To simplify or clarify those questions, you might assume f is a "typical" linear map in the sense of Lebesgue measure.)

In the 1980's Takens, Ruelle, Eckmann, Sano and Sawada extended this investigation to the typical images of attractors of dynamical systems. They asked when typical images are similar to the original. Now assume further that A is a compact invariant set for a map f on $\mathbf{R}^n$. When can we say that A and $f(A)$ are similar, based only on knowledge of the images in $\mathbf{R}^m$ of trajectories in A? For example, under what conditions on $f(A)$ (and the induced dynamics thereon) are A and $f(A)$ homeomorphic? Are their Lyapunov exponents the same? Or, more precisely, which of their Lyapunov exponents are the same? This talk (and corresponding paper) addresses these questions with respect to both the general class of smooth mappings f and the subclass of delay coordinate mappings.

Click here for a copy of the paper written with Will Ott.