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<b>1.</b> A child rolls a fair die six times in su she rolled each of the numbers 1–6 exac	

2. Calculate the number of ways of rearranging the letters CALBEARS so that  ${\tt E}$  appears somewhere to the left of  ${\tt S}.$ 

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<b>3.</b> How many ways are there to put 12 indistinguishable tacos into six distinguishable		12

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4. In how many ways can three students be assigned to six different discussion sections if each discussion section can receive at most one student?	

**5.** In how many ways can six students be assigned to three different discussion sections, if each discussion section must receive at least one student?

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**6.** For  $n \ge 0$ , let F(n) be the number of bit strings of length n that do not contain 00. We have F(0) = 1, F(1) = 2,  $F(2) = 3, \ldots$ 

**a.** For  $n \ge 2$ , explain why the number of bit strings of length n that start with 0 and do not contain 00 is equal to F(n-2).

**b.** For  $n \ge 1$ , explain why the number of bit strings of length n that start with 1 and do not contain 00 is equal to F(n-1).

**c.** Calculate the number of bit strings of length 4 that do not contain 00, using the insights of parts **a** and **b** (or otherwise, if you prefer).

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7. Before going on vacation for a week, you ask your spacey friend to water your ailing plant. Without water, the plant has a 90 percent chance of dying. With water, it has a 20 percent chance of dying. The probability that your friend will forget to water it is 30 percent.

**a.** What is the probability that your plant will be dead at the end of the week?

**b.** If the plant is dead when you return, what is the probability that your friend forgot to water it?

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1. Find an	explicit formula for $a_n$ (n	$\geq 0$ ) if	
	$a_n = a_{n-1} + 6a_{n-2}$ (for <i>n</i> )	$a \ge 2),  a_0 = -1,$	$a_1 = 22.$

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<b>2.</b> Find a function $y(t)$ $(t > 0)$ such that $ty' + 2y =$	$= 4t^2 \text{ and } y(1) = 0.$

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**3a.** A single card is to be drawn from a standard deck of 52 cards, examined and replaced. This procedure will be repeated four more times, for a total of five draws. What is the expected number of diamonds that will be drawn? [Note that there are 13 diamonds in the deck.]

**b.** If a 5-card poker hand is drawn at random from a standard deck of 52 playing cards, what is the expected number of diamonds in the hand?

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	(Scratchwork)

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4. Write as a sum of partial fractions	$\approx \frac{8x^3 + 2x^2 + 42x + 8}{(x-1)(x+1)(x^2+9)}.$

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**5a.** We suspect that a coin is biased so that it comes up heads 2/3 of the time. Flipping the coin 6000 times, we obtain 3929 heads and 2071 tails. To test the *null hypothesis* that the coin is biased in the way that we suspect, what  $\chi^2$  test statistic do we compute? (Your answer should be an explicit number, but you don't have to simplify it or estimate its value.)

**b.** After computing the test statistic, we obtain 0.0518 as the corresponding *p*-value. What should we conclude about the null hypothesis?

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**6.** "A company makes electric motors. The probability that an electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors?"

**a.** Calculate this probability (but don't bother to evaluate your answer as a decimal number).

**b.** What estimate for this probability would be supplied to us by the Poisson distribution?

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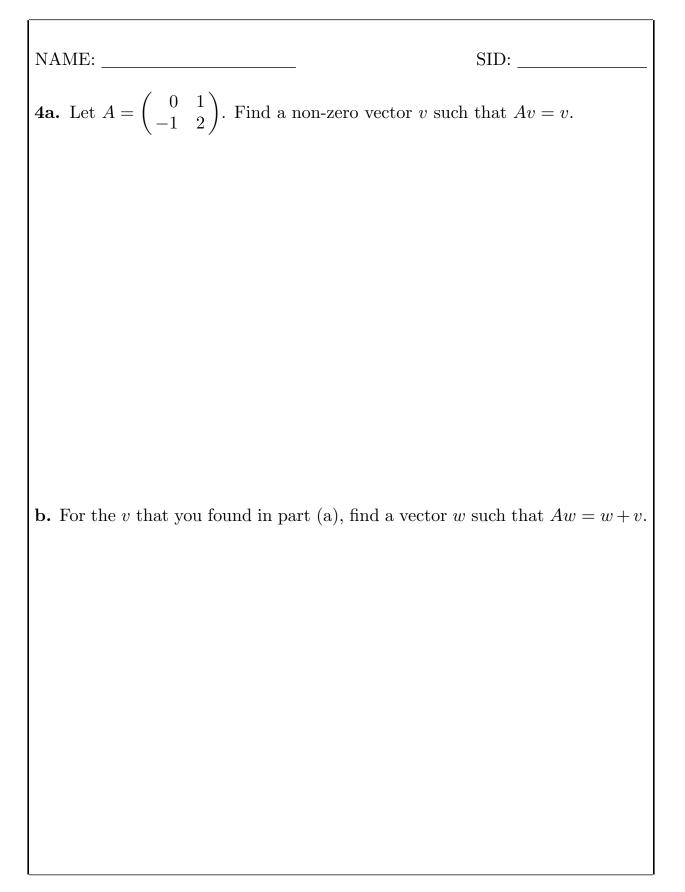
1. How many 5-card poker hands have at least two face cards? (The face cards are the jacks, queens and kings. Thus there are 12 face cards in a standard 52-card deck.)

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2. In some population, the probability that a woman has hemophilia is 0.5. If a woman has the hemophilia gene, then each of her sons independently has the probability 0.5 of having the disease; if she does not have the gene, then none of her sons will have the disease. None of Alice's three sons has hemophilia. What is the probability that Alice carries the hemophilia gene?

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<b>3.</b> Find all solutions to the differential equ $y(0) = y(\pi) = 10.$	uation $y'' - 4y' + 5y = 0$ that satisfy



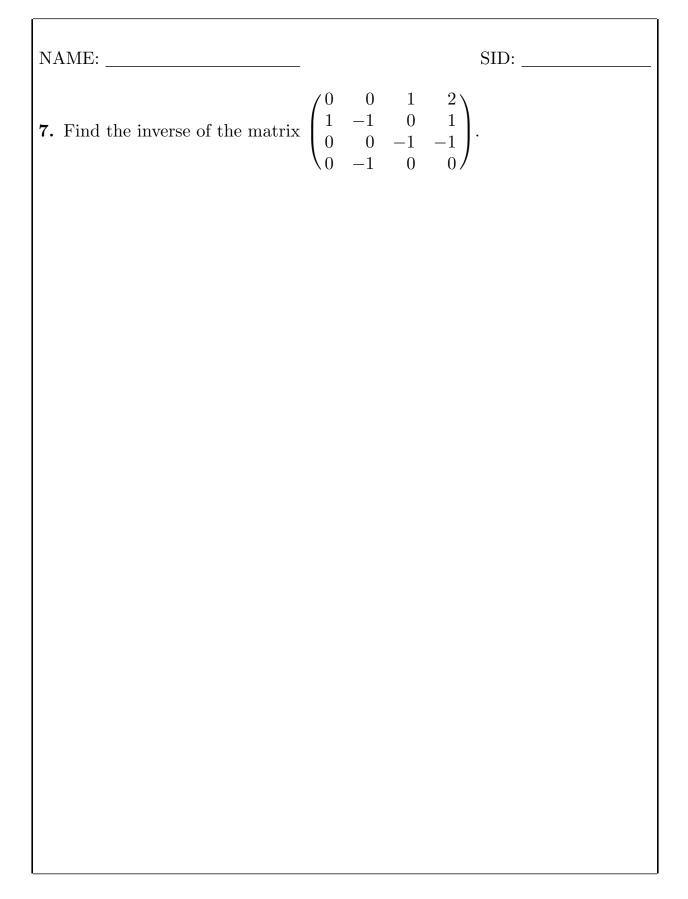
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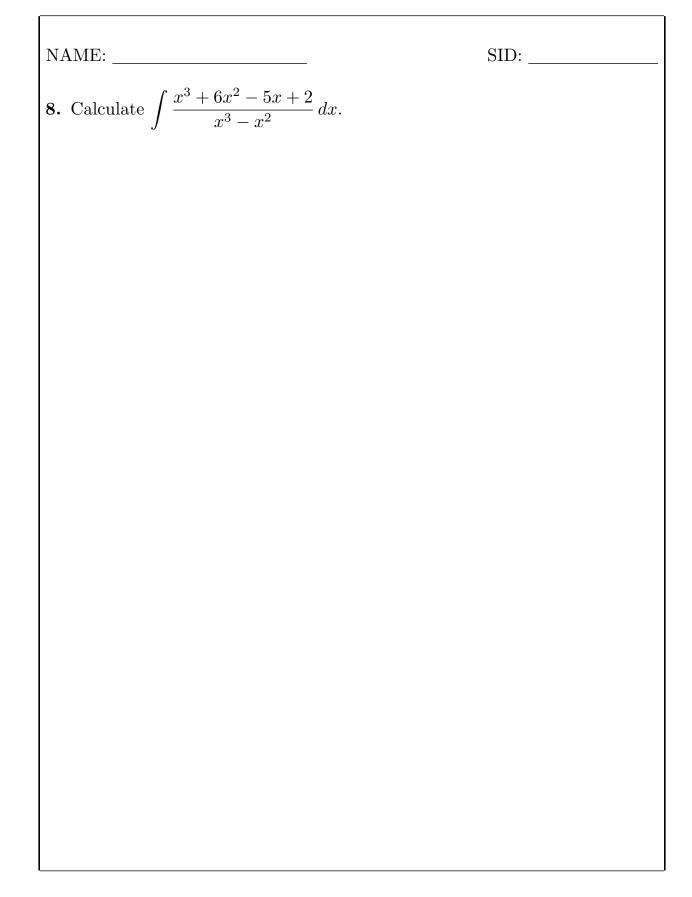
5. Suppose that A is an  $n \times n$  real matrix and that v is an eigenvector of A with eigenvalue  $\lambda$ . Suppose also that w is a vector of real numbers such that  $Aw = \lambda w + v$ . Show that  $y = e^{\lambda t}(tv + w)$  is a solution to the system of differential equations y' = Ay.

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6. The dials of RSF combination locks display numbers from 0 to 39. Combinations are lists of three numbers for which the differences between successive numbers are even, but not divisible by 4. Thus 30-4-10 is an allowable combination because 30 - 4 = 26 and 10 - 4 = 6 are even numbers but not multiples of 4. Other allowable combinations are 21-15-17, 0-10-0, 1-7-33; note that the three numbers in an allowable combination are either all odd or all even. Combinations that are *not* be permitted include 31-5-33 (because 33 - 5 = 28 is a multiple of 4) and 10-2-8 (because 10 - 2 = 8 is a multiple of 4).

Assume that RSF patrons are assigned locks randomly and independently, with all allowable combinations equally likely. Find the probability that the first ten patrons who requested locks today were given ten different combinations.





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<b>9.</b> The expansion of $(x + y + z)^3$	contains ten terms:
$(x+y+z)^3 = x^3 + 3x^2y + 3xy^2 + 3x$	$y^3 + 3x^2z + 6xyz + 3y^2z + 3xz^2 + 3yz^2 + z^3.$
How many terms are there in the	expansion of $(x + y + z + w)^{100}$ ?