1. (4 points) True or False (Fill in the blank with T or F)
   (a) If $A$ is an $n \times n$ matrix such that there is $b \in \mathbb{R}^n$ for which $Ax = b$ does not have a unique solution, then we can still apply Cramer’s rule to get at least one of the solutions. □
   (b) If $A$ is an $n \times n$ matrix, then $\text{Null}(A) \cap \text{Col}(A) = \{0\}$. □
   (c) Any real $n \times n$ matrix $A$ where $n$ is odd has a nonzero real eigenvector. □
   (d) There is an $n \times n$ real matrix with $n - 1$ distinct real eigenvalues and 1 complex eigenvalue with nonzero imaginary part. □

2. (4 points) Multiple choice: If $A$ is a $2 \times 2$ matrix such that $A^2 = I_2$ where $I_2$ is the $2 \times 2$ identity matrix, which of the following could be the characteristic polynomial of $A$? Mark an “x” in the box next to the correct answers:
   1. $(x - 1)(x - 1)(x + 1)$ □
   2. $x^2 - x$ □
   3. $x^2 - 2x + 1$ □
   4. $x^2 - 1$ □

3. (4 points) Multiple choice: Which of the following pairs of matrices are similar? Mark an “x” in the box next to the correct answers:
   1. $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $\begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$ □
   2. $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ □
   3. $\begin{pmatrix} 3 & 3 \\ 1 & 5 \end{pmatrix}$, $\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}$ □
   4. $\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$, $\begin{pmatrix} 5 & 3 \\ 1 & 1 \end{pmatrix}$ □
4. (6 points) An \( n \times n \) matrix \( O \) is called \textit{orthogonal} if \( O^T O = I_n \) where \( I_n \) is the \( n \times n \) identity matrix. Find a \( 4 \times 4 \) orthogonal matrix whose first column is

\[
\begin{pmatrix}
1 \\
-1 \\
2 \\
1
\end{pmatrix}
\]
Additional space for problem 4:
5. (6 points) Let $P_2$ be the set of polynomials of degree $\leq 2$. Consider the linear map $T : P_2 \rightarrow P_2$ given by $T(p)(x) = 2p(x) - p'(x)x$ where $p'$ denotes the derivative of the polynomial $p$.

1. Show that $T$ is linear (you can assume without proof that $(p_1 + p_2)'(x) = p_1'(x) + p_2'(x)$ where $p_1, p_2 \in P_2$).

2. Find the eigenvectors and eigenvalues of $T$. 
Additional space for problem 5:
6. (6 points) Find the determinant of

\[
A = \begin{pmatrix}
a & b & b & b & b \\
b & a & b & b & b \\
b & b & a & b & b \\
b & b & b & a & b \\
b & b & b & b & a \\
\end{pmatrix}
\]

where \(a\) and \(b\) are any real numbers (Hint: Use row operations to cancel many elements and simplify the calculation).
Additional space for problem 6:
7. (2 points) (Bonus) Show that if an orthogonal matrix $O$ has determinant $-1$, then $-1$ is an eigenvalue of $O$. 
Additional space for problem 7: