

Name: _____

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Your submission must consist of exactly the four pages given.

1. (4 points) True or False (Fill in the blank with T or F)
 - (a) If $AA^T = 2I_n$, then A is invertible.
 - (b) $A\mathbf{x} = \mathbf{b}$ has a solution if and only if the linear transformation corresponding to A is one-to-one.
 - (c) If A is a square matrix such that $A\mathbf{x} = \mathbf{b}$ is inconsistent for some \mathbf{b} , then the associated linear transformation T_A is not onto.
 - (d) There exist three vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^4$ such that every vector $\mathbf{b} \in \mathbb{R}^4$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.
2. (4 points) Multiple choice: Which of the following maps T are linear maps? Fill the correct answer(s) in the blank.
 1. $T : \mathbb{R} \rightarrow \mathbb{R}, T(x) = \sin^2(x) + \cos^2(x)$.
 2. $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ which is the rotation around the point $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ counterclockwise by $\frac{\pi}{3}$.
 3. $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, T\left[\begin{pmatrix} x \\ y \end{pmatrix}\right] = \begin{pmatrix} 3x - |y| \\ 2x + y \end{pmatrix}$.
 4. $T : \mathbb{R}^2 \rightarrow \mathbb{R}, T\left[\begin{pmatrix} x \\ y \end{pmatrix}\right] = 3x - 2y$.
3. (4 points) Multiple choice: Which of the following sets of vectors span \mathbb{R}^3 ? Fill the correct answer(s) in the blank.
 1. $\begin{pmatrix} 12 \\ 13 \\ 15 \end{pmatrix}, \begin{pmatrix} 16 \\ 17 \\ 19 \end{pmatrix}$
 2. $\begin{pmatrix} 5 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 11 \end{pmatrix}, \begin{pmatrix} 6 \\ 7 \end{pmatrix}$
 3. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$
 4. $\begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

4. (6 points) Find all real number(s) h such that the span of $\left\{ \begin{pmatrix} 1 \\ 2 \\ -6 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ h \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \right\}$ is \mathbb{R}^3

5. (6 points) Find the solution set of

$$\begin{pmatrix} 0 & 1 & 3 & -1 \\ 5 & 3 & -1 & 7 \\ 1 & 2 & 4 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

in parametric vector form.

6. (6 points) Determine if the following matrix is invertible. If so, find its inverse.

$$\begin{pmatrix} 0 & 2 & 1 \\ 1 & 4 & 2 \\ 2 & 5 & 3 \end{pmatrix}$$