Math 10A final exam, December 14, 2017

This was a 3-hour exam. Students had access to a one-page two-sided "cheat sheet" but were not allowed to bring in devices of any kind (e.g., phones, laptops, calculators, modern watches) or to have other written material.

The point counts for the nine problems were as follows:

Problem	1	2	3	4	5	6	7	8	9	Total
Points	8	9	7	9	9	6	8	6	8	70

1. Compute:

a. $\lim_{x\to\infty} \big(x - \ln(1+e^x)\big)$,

b. $\lim_{t\to\infty} \dfrac{t}{\sqrt{t^2-1}}$.

2. Explain carefully why the function

$$f(x) = \begin{cases} 0 & \text{if } x < 1 \\ \dfrac{1}{x^2} & \text{if } x \geq 1 \end{cases}$$

is a probabiltiy density function. Compute the corresponding cumulative distribution function (CDF) and sketch a graph of the CDF.

3. Which is more likely: getting 60 or more heads when a fair coin is tossed 100 times, or getting 175 or fewer heads when a fair coin is tossed 400 times?

4. For $z \geq 0$, let $T(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^z e^{-x^2/2}\,dx$. (Values of this frequently tabulated function appear on page 566 of our textbook.) Write in terms of T:

a. $P(X \geq 0)$ when X is normally distributed with mean -1 and standard deviation 2;

b. $P(-2 \leq X \leq 0.5)$ when X is normally distrbuted with mean 0 and standard deviation 1.

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5a. Find the number C so that

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ Ce^{-x^2/2} & \text{if } x \geq 0 \end{cases}$$

is a probability density function.

b. If X is a random variable whose PDF is $f(x)$, what is the expected value of X? (You can write your answer in terms of C or plug in the value of C that you found in part a.)

6. Find all numbers a and b such that

$$y(t) = ae^{bt}$$

is a solution to the differential equation

$$y''(t) = 3y'(t) - 2y(t).$$

7a. Find $\dfrac{dy}{dx}$ if $x = \dfrac{y}{1-y^2}$. (Your answer may involve both x and y.)

b. Evaluate the indefinite integral $\displaystyle\int x^2 \sin x \, dx$.

8. Let $f(x) = x^3 - 16x$, $a = -4$, $b = 2$. Find all numbers c with $a < c < b$ such that the line tangent to $y = f(x)$ at $(c, f(c))$ is parallel to the secant line between $(a, f(a))$ and $(b, f(b))$.

9a. Find a positive number C such that the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{a^n n!}$ converges for $a > C$ and diverges for $0 < a < C$.

b. Decide whether or not this series converges:

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{5} \cdots.$$

(Each term $\frac{1}{n}$ occurs n times.)

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