Math 10A midterm exam, October 27, 2016

1. Discuss the convergence of each of the following infinite series:

a. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{2^n}$,

b. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$,

c. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

2a. Express $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 - \frac{2i}{n}\right)\left(\frac{2}{n}\right)$ as an integral of the form $\displaystyle\int_0^1 f(x)\,dx$.

b. Express $\displaystyle\int_{-1}^{1} \cos x\,dx$ as a limit of Riemann sums.

3a. Find the indefinite integral $\displaystyle\int \sin^3 x\,dx$. (Use that $\sin^2 + \cos^2 = 1$.)

b. Evaluate $\displaystyle\int_0^{\infty} \frac{e^x}{(1+e^x)^2}\,dx$.

4. Calculate the volume of the football-shaped solid obtained by rotating the interior of the ellipse $\dfrac{y^2}{4} + \dfrac{x^2}{9} = 1$ about the x-axis.

5a. Evaluate this function of t: $\displaystyle\int_0^t se^{-s}\,ds$.

b. Find $\displaystyle\frac{d}{dt}\int_0^{t^2} e^{-x^2}\,dx$. (Hint: write the function to be differentiated in the form $G(t^2)$, where $G(u) = \displaystyle\int_0^u e^{-x^2}\,dx$.)

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.