Math 10A midterm exam, September 22, 2016

1. Find a formula for the derivative $\dfrac{dy}{dx}$ in each of the following three cases:

$$\textbf{a. } y = \sqrt[3]{\sin(x^3)}, \quad \textbf{b. } y = \ln\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right), \quad \textbf{c. } x^4 + 4y^4 = 7.$$

In the third case, your formula may involve y as well as x.

2. The equation $x^2 - x - 1 = 0$ has a root between 1.5 and 2.0. Suppose we employ Newton's method to find an approximation for the root, starting with 2 as our first guess for the root. What will the method output as our second guess? What will the method then output as our third guess?

3a. Let

$$f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Use the definition of *derivative* to express $f'(0)$ as a limit.

b. Decide whether or not the limit in part (a) exists; if it does, find its value.

4. Sketch the graph of $y = \dfrac{x}{1+x^2}$, taking note of such features as local maxima and minima, concavity, intercepts with the axes and behavior as $x \to \pm\infty$. Be sure to include the coordinates of any points where concavity changes or where the graph has a max or min.

5. Evaluate each of the following limits (if they exist):

$$\textbf{a. } \lim_{x \to 0+} x^{1/x}, \qquad \textbf{b. } \lim_{x \to \infty} \left(\sqrt{x^2 + x + 1} - x\right).$$

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.