

Mathematics Department Colloquium

Organizer: Maciej Zworski

Thursdays, 4:10–5:00pm, 60 Evans

Sept 29 **F. Alberto Grünbaum**, UC Berkeley

Differential operators with matrix coefficients: a collection of open problems in algebra and (maybe) non-commutative algebraic geometry

The study of sets differential operators acting on scalar valued functions and sharing lots of common eigenfunctions has very strong connections with algebraic curves and vector bundles on them. This goes back to Burchnell and Chaundy (1921). The first look at these issues seems to go back to I. Schur (1905), who showed that the commutator of one ordinary differential operator is necessarily commutative.

The work of Krichever on soliton type equations, like KdV and KP, has brought renewed interest in this connection which leads to explicit solutions in terms of theta functions. This is all related to Sato Grassmanians and loop groups, as in the work of Segal and Wilson. This has led to recent work by Berest, Wilson and Kontsevich (among others) dealing with the Weyl algebra.

I consider a similar problem for differential operators with matrix coefficients, acting on matrix valued functions, and having a “matrix valued eigenvalue”. The corresponding algebra is now (in general) non-commutative.

I will describe ab-initio a few examples where I can compute explicitly a set of generators and relations for the algebra in question. All my work is “bare-hands” (with computer assistance) and could benefit from insights coming from “noncommutative algebraic geometry”. I expect to get advice and suggestions from the audience.