Department of Mathematics, University of California, Berkeley

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1999 Chern lectures
September 16-October 6, 1999

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Department of Mathematics Colloquium

Counting rational points and rational curves: from Waring's problem to quantum cohomology

Thursday, September 16, 4:10-5:00pm

Sibley Auditorium, Bechtel Hall

Reception in 1015 Evans following the lecture

Minicourse

Moduli spaces, Frobenius manifolds, and quantum cohomology

Wednesdays and Fridays
September 17/22/24/29, October 1
2:10-3:00pm, 3 LeConte

Monday, October 4
2:10-3:00pm, 50 Birge
Wednesday, October 6
2:10-3:00pm, 3 LeConte

Abstract

Consider a homogeneous equation of degree d with integer coefficients $F(x_0,\dots,x_r)=0$, and ask the following question: how many integer solutions are there with all $|x_i|$ at most B, when B is large? A similar question can be asked about the variety of solutions whose coordinates are homogeneous polynomials in two variables of bounded degree. An elementary heuristic argument relying upon a probabilistic reasoning in the first case, and count of constants in the second case, suggests that the answer must depend on the sign of the number $k=r+1-d$: there must be `many' solutions for positive $k$, `few' for negative k, and some interesting boundary effects might take place for $k=0$.

In fact, k is simply the degree of the first Chern class of the projective manifold $F=0$ (if it is nonsingular), and it turns out that many results of number theory and algebraic geometry nicely fit into this crude heuristic scheme, if one makes some subtle changes in basic definitions and questions.

The first part of the colloquium talk will discuss the number-theoretical program, which can be considered as an extension of the classical work using the circle method.

The second part of the colloquium talk will be dedicated to the counting of rational curves. Motivated by quantum string theory, this subject has developed into a rich and beautiful theory centered around quantum cohomology and the mirror conjecture. In the introductory lecture, the physical context of quantum cohomology will be described. The following minicourse will contain a more detailed review of the relevant mathematical constructions and results.

Program for the minicourse

1
Geometry of the moduli spaces of stable curves
Friday, September 17, 2:10-3:00pm

2
Language of operads, modular operads
Wednesday, September 22, 2:10-3:00pm

3
Algebras over the homological modular operad

and formal Frobenius manifolds
Friday, September 24, 2:10-3:00pm

4
Frobenius manifolds and integrable differential equations
Wednesday, September 29, 2:10-3:00pm

5
Quantum cohomology of algebraic manifolds
Friday, October 1, 2:10-3:00pm

6
Floer Memorial Lecture
Frobenius manifolds from Batalin-Vilkovisky algebras
Monday, October 4, 2:10-3:00pm
50 Birge

7
Unfolding of singularities and Saito's frameworks
Wednesday, October 6, 2:10-3:00pm

All lectures of the minicourse
with the exception of nr. 6
will be held in 3 LeConte
