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Spring 1994, Math 114
First Midterm Exam

28 February, 1994
2:10-3:00 PM

1. (25 points) Suppose $L:K$ is a field extension, and $\alpha \in L$ is algebraic over K . Let $f \in K[t]$ be of least degree among all nonzero polynomials over K having α as a root (the “minimal polynomial” of f).
 - (a) (8 points) Show f is irreducible.
 - (b) (17 points) Show that every polynomial $h \in K[t]$ satisfied by α is a multiple of f .

2. (25 points) Suppose $L:K$ is a field extension, and $\alpha, \beta \in L$ are elements algebraic over K , of degrees m and n respectively. Show that $\alpha + \beta$ is algebraic over K , and obtain an upper bound on its degree over K , in terms of m and n .

3. (25 points) (In parts (a) and (c) below, if you do not remember the definition, but give a condition which Stewart proves equivalent to the named condition, you will get partial credit.)
 - (a) (6 points) Define what it means for an algebraic extension $L:K$ to be *normal*.

 - (b) (3 + 3 points) Give an example of a nontrivial algebraic extension $L:K$ (i.e., an algebraic extension with $L \neq K$) that is normal, and an example of one that is not normal. (You do not have to prove that your examples have the indicated properties.)

 - (c) (6 points) Define what it means for an algebraic extension $L:K$ to be *separable*.

 - (d) (3 + 4 points) Give an example of a nontrivial algebraic extension that is separable, and an example of one that is not separable. (You do not have to prove that your examples have the indicated properties.)

4. (25 points) Suppose $M:K$ is a finite normal algebraic extension, and L is a subextension of M (a subfield of M containing K), which is carried into itself by every K -automorphism of M . Show that $L:K$ is also normal.