SPRING 2009. FINAL EXAM
MATH 121B

Your name

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Student ID number

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1	2	3	4	5	6	7	8	total

1. A particle moves without friction under gravity on the surface of the paraboloid $z = x^2 + y^2$. Write the Lagrange equations of motion in cylindrical coordinates.

2. Prove the identity

$$\int_0^\infty \frac{y^m}{(1+y)^{n+1}}dy = \frac{1}{(n-m)C(n,m)},$$

for all positive integers n and m, $n > m$.

3. Solve the differential equation

$$(1 - x^2)y'' + xy' + 3y = 0.$$

4. Find the steady-state temperature inside the hemisphere

$$x^2 + y^2 + z^2 \leq 1, z \geq 0,$$

if its curved surface is held at

$$u = \cos^3 \theta,$$

and the equatorial plane at $u = 0$.

5. An infinite string occupying the x-axis $x \geq 0$ has a fixed left end. Find the vertical displacement as a function of x and t if initial displacement is given by function xe^{-x} and initial velocity is zero. (Hint: this problem can be solved by use of an integral transform or the formula $y(x, t) = f(x + vt) + g(x - vt)$. You can use either of these methods.)

6. Find the characteristic frequencies and normal modes of the membrane in the shape of a quarter circle of radius 1

$$x \geq 0, y \geq 0, x^2 + y^2 \leq 1.$$

(Hint: your answer should involve zeros of Bessel functions.)

7. There are 4 red, 2 white and 3 blue balls in the first box and 6 red and 5 white balls in the second box. You select a box at random and from it pick a ball at random. If a ball is white, what is the probability that it came from the first box?

8. 10 particles are put in 10 boxes. Find the probability that each box is occupied according to three different kinds of statistics (Maxwell-Bolzmann, Fermi-Dirac and Bose-Einstein).