

Mathematics H185  
Sarason

March 3, 2006

### MIDTERM EXAMINATION 1

**Open book, open notes.** In your proofs, you may use any results from Chapters I–V of the textbook, including results from the assigned exercises in those chapters. Be clear about what results you use.

The points for each problem are in parentheses.

- (10) Find all complex numbers  $c$  such that  $\sin c$  is real.
- (10) Let the linear-fractional transformation  $\varphi$  satisfy  $\varphi(1) = -1$  and  $\varphi(-1) = 1$ . Prove  $\varphi \circ \varphi$  is the identity map.
- (20) (a) Prove that the power series  $\sum_{n=1}^{\infty} \frac{z^{(2^n)}}{n}$  has radius of convergence 1.  
  
(b) Let  $f$  be the function in the unit disk represented by the power series in (a). Let  $\omega$  be a  $2^k$ -th root of unity ( $k = 1, 2, \dots$ ). Prove  $\lim_{r \rightarrow 1} f(r\omega) = \infty$ .
- (10) Let the function  $f$  be holomorphic in the unit disk  $\mathbb{D}$ . Assume there is a polynomial  $p$  of positive degree such that  $p \circ f$  is real valued. Prove  $f$  is constant.