April 14, 2004 Math 118 M. Rieffel

Second Midterm Exam

State your answers clearly and fully (with whole sentences, please). Include all your work.
(Total points = 40.)

1. For each integer n let V_n and W_n be the usual subspaces of $L^2(R)$ coming from the Haar scaling function and wavelet (for dilation by 2).

6 a) Explain carefully how the V_n 's and W_n 's are obtained.

6 b) Exhibit explicitly the Haar wavelet orthonormal basis for W_2 .

c) For each n let X_n be the subspace of W_n consisting of
8 those functions which have value 0 outside the interval $[0, 5]$. Determine the dimensions of X_1 and X_{-1} by finding bases for them. Justify your answer.

2. Fix a positive integer N , and let x and y be complex-valued functions on $\{0, 1, ..., N-1\}$.

5 a) Define what is meant by the (discrete) Fourier transform, $\hat{x}$, of x.

5 b) Define the convolution $x*y$ of x and y.

10 c) Show that $(x*y)^\wedge = \hat{x}\,\hat{y}$, the pointwise product.