Prof. Bjorn Poonen
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MATH 185 MIDTERM 2

Do not write your answers on this sheet. Instead please write your name, your student ID, and all your answers in your blue books. Total: 100 pts., 50 minutes.

(1) (5 pts. each) For each of (a)-(c) below: If the statement is true, write TRUE. If the statement is false, write FALSE. (Please do not use the abbreviations T and F, since in handwriting they are sometimes indistiguishable.) No explanations are required in this problem.

(a) If a function $f(z)$ is holomorphic on an open subset U of $\mathbb{C}$, and γ is a closed piecewise C^1 curve contained in U, then $\int_\gamma f(z)\,dz = 0$.

(b) If f is an entire function, and $\gamma\colon [0,1] \to \mathbb{C}$ and $\Gamma\colon [0,1] \to \mathbb{C}$ are piecewise C^1 curves such that $\gamma(0) = \Gamma(0)$ and $\gamma(1) = \Gamma(1)$, then $\int_\gamma f = \int_\Gamma f$.

(c) Let $\mathbb{C}^*$ be the set of nonzero complex numbers, and let f be a holomorphic function on $\mathbb{C}^*$. If f is bounded on $\mathbb{C}^*$, then f is constant.

(2) (10 pts. each) Evaluate the following integrals:

(a) $\displaystyle\int_{|z|=3} \overline{z}\,dz$

(b) $\displaystyle\int_{|z|=1} \frac{2z-1}{z^2-8z+15}\,dz$

(3) Let $f(z) = \dfrac{5z+3}{e^z-1-z}$, defined on the open subset of $\mathbb{C}$ where the denominator is nonzero.

(a) (15 pts.) What kind of singularity does $f(z)$ have at $z=0$?

(b) (25 pts.) Compute the residue of $f(z)$ at $z=0$.

(4) (25 pts.) For $R > 0$, let γ_R be the upper half of the circle $|z| = R$, taken counterclockwise. (Thus γ_R is a semicircular path from R to $-R$.) Prove that

$$\lim_{R\to+\infty} \left| \int_{\gamma_R} \frac{e^{iz}}{z^2}\,dz \right| = 0.$$

This is the end! At this point, you may want to look over this sheet to make sure you have not omitted any problems. Please take this sheet with you as you leave.