Exam 2 MATH 185 Spring 2001 Prof. Croot

Show all your work.

1. Evaluate

$$\int_C \frac{\sin z}{z^3(z-1)} dz,$$

where C is the positively oriented simple closed contour $|z| = 1/2$.

2. Prove that

$$\int_C \frac{\cos z}{\sin^2 z} dz = 0,$$

where C is any simple closed contour not passing through a zero of $\sin z$.

3. Prove Liouville's Theorem: if $f(z)$ is entire and $|f(z)|$ is bounded, then $f(z)$ is a constant function.

4. Suppose $f(z)$ is analytic for $|z| < 1$, and $|f^{(n)}(0)| \leq 1$, for all $n = 0, 1, 2....$ Show that there is an entire function $g(z)$ such that $g(z) = f(z)$ for $|z| < 1$. (Justify any claims about convergence).

5. Find the Laurent series for

$$f(z) = \frac{z^2+1}{z(z-3)}.$$

in the annuli $0 < |z| < 3$ and $3 < |z| < \infty$.