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MATH 140 MIDTERM #2

Do not write your answers on this sheet. Instead please write your name, your student ID, and all your answers in your blue books. Total: 100 pts., 50 minutes.

(1) (5 pts. each) For each of (a)-(c) below: If the proposition is true, write TRUE. If the proposition is false, write FALSE. (Please do not use the abbreviations T and F, since in handwriting they are sometimes indistiguishable.) No explanations are required in this problem.

(a) If S is a regular surface in $\mathbb{R}^3$, and p is a parabolic point on S, then all points of S lie on one side of the tangent plane $T_p(S)$. (By "one side", we mean one of the closed half-spaces formed when $\mathbb{R}^3$ is cut by $T_p(S)$.)

(b) If a straight line L is contained in a regular surface S, then at any point $p \in L$, the direction given by L is a principal direction of S at p.

(c) If C is a regular curve in the right open half of the xz-plane, and C is covered by a single parametrization $\alpha\colon I \to C$, then the surface of revolution obtained by rotating C all the way around the z-axis is orientable.

(2) (20 pts.) Let U be an open subset of $\mathbb{R}^2$, and let $\mathbf{x}\colon U \to \mathbb{R}^3$ be a parametrization of a regular surface S. Let $q \in U$, and let $p = \mathbf{x}(q)$. Suppose that the coefficients of the first fundamental form for this parametrization at the point $q \in U$ are $E = 1$, $F = 3$, and $G = 18$. Let β_1, β_2 be regular parametrized curves in U such that $\beta_1(0) = \beta_2(0) = q$ and $\beta_1'(0) = (1,0)$ and $\beta_2'(0) = (1,1)$. Find $\cos\theta$, where θ is the angle between the images of these two curves in S at p.

(3) (25 pts.) Let S be the cylinder given by the equation $x^2 + y^2 = 4$ (and no condition on z) in $\mathbb{R}^3$. Orient S by equipping it with the *outward* unit normal field. Let $p = (2,0,0) \in S$, and let $v = (0, \sqrt{3}/2, 1/2) \in T_p(S)$. What is the normal curvature of S at p along the direction of v?

(4) Let C be a regular curve contained in the unit sphere S^2, and suppose that $\alpha\colon (0,L) \to C$ is a parametrization of all of C by arc length. (Thus the total arc length of C equals L.) Suppose also that $0 < L < 2\pi$. Connect each point on C to the center of the sphere with a line segment, but exclude the endpoints. The union of these "open" line segments forms a regular surface S_1 contained in the open unit ball.

Let S_2 be the open subset of the plane described in polar coordinates (r,θ) by the inequalities $0 < r < 1$ and $0 < \theta < L$. (Thus S_2 is the interior of a sector of a unit disk: if L is small, it looks like a piece of pizza.)

(a) (30 pts) Prove that S_1 and S_2 are isometric. (In doing this problem, you may write down and use parametrizations without proving that they are parametrizations, as long as they really are parametrizations!)

(b) (10 pts) What is the total area of S_1?

This is the end! At this point, you may want to look over this sheet to make sure you have not omitted any problems. Check that your answers make sense! Please take this sheet with you as you leave.