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Spring 2004, Math 185, Sec. 1
First Midterm

23 Feb., 2004
10:10-11:00

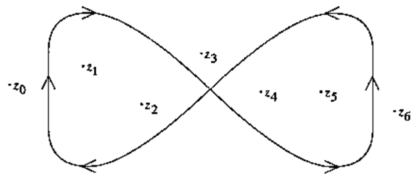
1. (27 points, 9 points each.) Find the following. Correct answers, with or without work, will give full credit. If answers are incorrect, partially correct work may bring partial credit.

(a) The radius of convergence of the power series $\Sigma\,(5\,i)^n z^{2n}$.

(b) $\mathrm{Log}\ e^{1+4i}$

(c) The winding numbers of the path γ shown below, (understood to trace out the indicated curve exactly once in the direction shown by the arrows) about each of the labeled points.

(If you think this question would take you more than 5 minutes, you're probably using too hard a method. In that case, fill in for partial credit any parts you can do quickly, and leave the rest to do at the end if you have time.)



2. (30 points, 10 points each.) Complete the following definitions. Unless otherwise stated you may use without defining them any terms or symbols which our text defines before it defines the concept asked for. You do not have to use exactly the same words as Stewart and Tall, but for full credit your statements should be clear, and mean the same as theirs. Something that was *proved* equivalent to the concept in question is not the same as the definition.

(a) A subset S of $\mathbb{C}$ is said to be *path-connected* if

(b) If $\gamma\colon [a,b] \to \mathbb{C} \setminus \{0\}$ is a path, and $\theta\colon [a,b] \to \mathbb{R}$ is a continuous choice of argument for γ, then $w(\gamma, 0)$, the *winding number* of γ about 0, is defined to be

(c) A subset $D \subseteq \mathbb{C}$ is called a *star domain* if

3. (30 points, 10 points each.) For each of the items listed below, either *give an example* with the properties stated, or give a brief reason why *no such example exists.*

If you give an example, you do *not* have to prove that it has the property stated; however, your examples should be specific; i.e., even if there are many objects of a given sort, name a particular one. If you give a reason why no example exists, don't worry about giving a detailed proof; the key relevant fact will suffice.

(a) A continuous function f on $\mathbb{C}$ and a closed contour γ in $\mathbb{C}$ such that $\int_\gamma f \neq 0$.

(b) A differentiable function f on $\mathbb{C} \setminus \{0\}$ and a closed contour γ in $\mathbb{C} \setminus \{0\}$ such that $\int_\gamma f \neq 0$.

(c) A differentiable function f on the disk $D = \{z \in \mathbb{C} \mid |z| < 1\}$ and a closed contour γ in D such that $\int_\gamma f \neq 0$.

4. (13 points) Suppose $u, v, w\colon \mathbb{C} \to \mathbb{R}$ are functions such that, if we define $f(z) = u(z) + i\,v(z)$ and $g(z) = u(z) + i\,w(z)$, the functions $f, g\colon \mathbb{C} \to \mathbb{C}$ are both differentiable. Prove that the function $v - w$ is constant.

(You may use results proved in the text. In doing so, make clear what facts you are calling on. No need to specify what numbers the text gives them.)