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Math 1B, Final Examination

Section 1, 11-12am, N.Reshetikhin, May 19, 2003

Student's Name:

TA's name:

Student's i.d. number:

1.10 pnts Evaluate the integral

$$\int_0^1 x^3 \sqrt{1+x^2} dx$$

2.10 *pnts* Evaluate the integral

$$\int x^2 \ln(1+x) dx$$

3.15 *pts* Indicate which of the following statements are true and which are false. Do not show your work.

1. $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$ converges by comparison test with $\int_1^{\infty} \frac{1}{x^2} dx$.

2. $\int_1^{\infty} \frac{\sin^2 x}{x} dx$ diverges by comparison test with $\int_1^{\infty} \frac{1}{x} dx$.

3. $\int_0^2 \frac{dx}{(x-1)^2}$ is a convergent improper integral.

4. $\int_0^{\infty} \frac{1}{x^2} dx$ is a divergent improper integral.

5. $\int_0^{\infty} \frac{e^{-x}}{\sqrt{x}} dx$ is a convergent improper integral.

4.15 *pts* Find the radius and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \ln(n) x^n$$

5.15 *pnts* State whether the following series is absolutely convergent, conditionally convergent, or divergent. Do not show your work.

1. $\sum_{n=1}^{\infty} (-1)^n.$

2. $\sum_{n=1}^{\infty} \cos(\pi n) \frac{1 + 7n^2}{3n + 14n^3}.$

3. $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n}).$

4. $\sum_{n=1}^{\infty} \frac{ne^n}{n!} (-1)^n.$

5. $\sum_{n=1}^{\infty} \cos(\pi n) \frac{1}{n \ln(n)}$

6.15 *pnts* For each statement indicate whether it is true or false. Do not show your work.

1. If $\sum_{n=1}^{\infty} c_n$ converges, then $\sum_{n=1}^{\infty} (-1)^n c_n$ also converges.
2. If $f(x) > 0$ is monotonically decreasing and $\int_{1000}^{\infty} f(x) dx < \infty$ then $\sum_{n=1}^{\infty} f(n)$ converges.
3. If the sequence $\{a_n\}$ converges and the sequence $\{b_n\}$ diverges then $\{a_n + b_n\}$ diverges.
4. If the sequence $\{a_n\}$ converges and the sequence $\{b_n\}$ diverges then $\{a_n b_n\}$ diverges.
5. If $\sum_{n \geq 0} a_n 5^n$ converges and $\sum_{n \geq 0} a_n (-6)^n$ diverges, then $\sum_{n \geq 0} a_n 8^n$ diverges.

7.15 *pnts* Indicate whether each of the following series converges absolutely, converges conditionally, or diverges. Do not show your work.

1.
$$\sum_{n=1}^{\infty} \frac{n}{(n+1)^3}$$

2.
$$\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^3}$$

3.
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{(n+1)^3}$$

4.
$$\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right) \frac{n^2}{(n+1)^3}$$

5.
$$\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right) \frac{n^4}{(n+1)^3}$$

8.15 *pts* Find the general solution to the differential equation

$$xy' - y = x .$$

9.15 *pnts* Solve the initial-value problem

$$\frac{dy}{dx} = 1 + x^2 + y + x^2y, \quad y(0) = 0 .$$

10.15 *pnts* Find the solution to the initial-value problem

$$y'' - y = e^x, \quad y(0) = 0, \quad y'(0) = 1 .$$

11.15 *pnts* Find the general solution to the differential equation:

$$y'' - 2y' - 3y = x .$$

12.20 *pnts* Find the power series solution to the differential equation:

$$y'' - xy = 0, \quad y(0) = 0, \quad y'(0) = 1 .$$