George M. Bergman
959 Evans

Spring 2001, Math 53M
First Midterm – Make-up Exam

23 February, 2001
10:10-11:00 AM

1. (54 points, 9 points apiece) Find the following. If an expression is undefined, say so.

(a) dy/dx, where $x = 2\sin(e^t)$, $y = 5\cos(e^t)$. Express your answer as a function of t.

(b) The length of the space curve given by the parametric equations $x = 2e^t$, $y = e^{2t}$, $z = t$ $(-1 \le t \le +1)$.

(c) $\lim_{(x,y)\to(0,0)} (|x|+2)/(|y|+7)$.

(d) The equation of the plane tangent to the surface $z = (x^2+y)^{1/2}$ at the point where $x = 3$, $y = 7$.

(e) $\frac{\partial^2}{\partial x \partial y} f(xy^2)$ where f is a differentiable function. (Express your answer in terms of f and its derivatives.)

(f) $\int_0^1 (t^2 \times (t^2\mathbf{i} + e^{-t^2}\mathbf{j} + (\tan t)\mathbf{k}))\,dt$ (where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the standard basis vectors in $\mathbb{R}^3$).

2. (34 points) **(a)** (20 points) Let f be a positive continuous real-valued function on the interval $[-\pi/4, \pi/4]$. Let A denote the area between the curve whose expression in polar coordinates is $r = f(\theta)$ $(-\pi/4 \le \theta \le \pi/4)$ and the two lines $\theta = -\pi/4$ and $\theta = \pi/4$. Let B denote the area between the curve whose expression in polar coordinates is $r = f(\theta/2)$ $(-\pi/2 \le \theta \le \pi/2)$ and the vertical axis $\theta = \pm\pi/2$. Show that $B = 2A$. You may assume area formulas given in Stewart.

(b) (14 points) Find the area between the y-axis and the curve whose expression in polar coordinates is $r = \sec\theta/2$. You may use the result of part (a) whether or not you have proved it; or you may use any other method that gives the correct answer.

3. (12 points) Find equations in *Cartesian* (i.e., (x, y, z)) and *spherical* coordinates for the surface described in cylindrical coordinates by the equation $r^2 = z^2 + 1$.