Mathematics Department Colloquium

Organizer: Kenneth Ribet

Thursday, 4:10–5:00pm, 60 Evans

Jan. 24 **Samit Dasgupta**, Harvard University

A p-adic approach to Hilbert's 12th problem

It is well known that the square root of any integer can be written as a linear combination of roots of unity. A generalization is the "Kronecker-Weber Theorem," which states that in fact any element that generates an abelian Galois extension of the field of rational numbers $\mathbf{Q}$ can also be written as such a linear combination. The roots of unity may by viewed as the special values of the analytic function $e(x) = \exp(2\pi ix)$ where x is taken to be a rational number. Broadly speaking, Hilbert's 12th problem is to find an analogous result when $\mathbf{Q}$ is replaced by a general algebraic number field F, and in particular to find the analytic functions that play the role of $e(x)$ in this general setting.

Hilbert's 12th problem has been solved in the case where F is an imaginary quadratic field, with the role of $e(x)$ being played by certain modular forms. All other cases are, generally speaking, unresolved. In this talk I will discuss the case where F is a real quadratic field, and more generally, a totally real field. I will describe relevant conjectures of Stark and Gross, as well as current work using a p-adic approach and methods of Shintani. A proof of these conjectures would arguably provide a positive resolution of Hilbert's 12th problem in these cases.