UC Berkeley Math 10B, Spring 2014: Final exam

Prof. Persson, May 12, 2014

Name: ______________________

SID: ______________________

Neighbors: Please write the names of the students next to you:

Left: ______________________

Right: ______________________

Section: Circle your discussion section below:

Time	Sec	Room	GSI
MWF 8-9am	102	4 Evans	Jason Ferguson
MWF 9-10am	103	41 Evans	Jason Ferguson
MWF 10-11am	104	39 Evans	Anna Lieb
MWF 11-12pm	105	39 Evans	Anna Lieb
MWF 12-1pm	106	41 Evans	Zvi Rosen
MWF 1-2pm	101	45 Evans	Zvi Rosen
MWF 2-3pm	107	3113 Etcheverry	Ralph Morrison
MWF 3-4pm	108	103 Moffitt	Ralph Morrison

Other/none, explain: ______________________

Grading

1	/ 6
2	/ 4
3	/ 3
4	/ 6
5	/ 6
6	/ 4
7	/ 4
8	/ 5
9	/ 6
10	/ 6
11	/ 5
12	/ 4
13	/ 6
	/65

Instructions:

- Closed book: No notes, no books, no calculators.
- Exam time 180 minutes, do all of the problems.
- You must justify your answers for full credit.
- Write your answers in the space below each problem.
- If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.
- Unless we ask for an actual number, we will accept answers in terms of any combination of [finite] sums, differences, products, quotients, polynomials, exponents, logs, absolute values, trig functions, inverse trig functions, factorials, $P(n,k)$, $C(n,k)$, $S(n,k)$, and $p_k(n)$.

1. a) (3 points) In how many ways can 6 employees be assigned to 3 different (distinguishable) committees, if each committee must have at least one member?
Compute the actual number!

b) (3 points) Same problem, but one of the members of each committee must be designated the chair of the committee.
No simplification needed!

2. (4 points) Suppose a final exam has 13 questions (distinguishable) and a total score of 65 points. Each problem is worth an integer number of points, and each problem must be worth at least 3 points. In how many ways can the points be assigned?

3. (3 points) If X is a binomial distribution with $E[X] = 3$ and $\text{Var}[X] = 2$, then compute the number of trials n and the probability of success p in X.

4. A coin having probability 0.8 of landing on heads is flipped. Alice observes the result–either heads or tails–and rushes off to tell Bob. However, with probability 0.4, Alice will have forgotten the result by the time she reaches Bob. If Alice has forgotten, then, rather than admitting this to Bob, she is equally likely to tell Bob that the coin landed on heads or that it landed tails. (If she does remember, then she tells Bob the correct result.)

a) (2 points) What is the probability that Bob is told that the coin landed on heads?

b) (2 points) What is the probability that Bob is told the correct result?

c) (2 points) Given that Bob is told that the coin landed on heads, what is the probability that it did in fact land on heads?

5. You toss a coin and roll a die (both possibly biased), and repeat this experiment for a total of 100 times. You record the joint outcomes in the following table:

	1	2	3	4	5	6
H	16	18	5	8	10	2
T	10	6	5	3	10	7

Suppose you want to test the null hypothesis H_0 that the coin toss and the die roll are independent.

a) (3 points) Construct a table showing the expected frequencies under the null hypothesis H_0.

b) (3 points) What is the χ^2-statistic for this data? Describe how to use this to determine if the null hypothesis can be rejected.

6. (4 points) Evaluate the integral $\displaystyle\int \frac{x-4}{x^3+4x}\,dx$.

7. (4 points) Find all solutions, if any, to the differential equation $t^{3/2}y' = e^{-y}$.

8. (5 points) Find all real values ω such that the boundary value problem

$$y'' + 2\omega y' + 5\omega^2 y = 0, \quad y(0) = 0 \text{ and } y(\pi) = 0,$$

has a non-trivial real-valued solution (that is, a solution other than $y(t) = 0$).

Hint: First find the general solution to the differential equation, which will involve the unknown constant ω. Then try to make the solution fit the boundary conditions.

9. Consider the initial value problem $y' - y = -2t$, $y(0) = 1$.

a) (3 points) Use Euler's method:

$$y_{n+1} \approx y_n + hf(t_n, y_n) \quad (n = 0, 1, \ldots, N-1).$$

with $h = 1/2$ compute an approximate value of $y(1)$.

b) (3 points) Use the *midpoint method*, defined by:

$$y_{n+1} \approx y_n + hf\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}f(t_n, y_n)\right) \quad (n = 0, 1, \ldots, N-1).$$

with $h = 1$ to compute another approximate value of $y(1)$.

10. Consider the two matrices A and B below.

$$A = \begin{pmatrix} 7 & 4 & 2 & 1 \\ 2 & 8 & 4 & 2 \\ 4 & 4 & 8 & 4 \\ 1 & 4 & 2 & 7 \end{pmatrix}, \qquad B = \frac{1}{12}\begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & 0 & 2 \end{pmatrix}$$

a) (3 points) Compute the product AB.

b) (3 points) Solve the linear system $Ax = (1, 2, 3, 4)^T$. *Hint*: Use the answer in **a)**.

11. (5 points) Consider the linear system

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & -1 & a \\ 3 & a & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3+a \\ 1 \\ 2 \end{pmatrix}$$

Find the values of a, if any, such that the system has **(a)** a unique solution, **(b)** an infinite number of solutions, and **(c)** no solutions.

12. (4 points) Find the least square line $y = \beta_0 + \beta_1 x$ for the data points

$$(x, y) = (0, 1),\ (1, -1),\ \text{and}\ (2, 0).$$

13. Consider the matrix

$$A = \begin{pmatrix} 12 & 1 & 4 \\ 2 & 11 & 4 \\ 1 & 3 & 7 \end{pmatrix}$$

a) (3 points) Show that three eigenvectors are given by

$$x_1 = (3, -2, -1)^T, \quad x_2 = (2, 2, 1)^T, \quad \text{and} \quad x_3 = (1, 1, -2)^T,$$

and find the corresponding eigenvalues.

b) (3 points) Solve the initial value problem $y' = Ay$, $y(0) = (5, 0, 0)^T$.

(Scratch Paper, Page 1)

(Scratch Paper, Page 2)