For a complex number τ with positive imaginary part, we consider the quotient $E_\tau := \mathbf{C}/(\mathbf{Z}+\mathbf{Z}\tau)$. This quotient can be embedded in the complex projective plane via the Weierstrass P-function and its derivative. The image of E_τ in the projective plane is then a complex elliptic curve given by an equation of the form $y^2 = x^3 + a_\tau x + b_\tau$. To this equation one associates the complex number

$$j(\tau) := 1728 \frac{4a_\tau^3}{4a_\tau^3 + 27b_\tau^2},$$

the so-called j-invariant of E_τ.

The j-invariant classifies complex elliptic curves up to isomorphism: E_τ is isomorphic to $E_{\tau'}$ if and only if $j(\tau) = j(\tau')$. The complex analytic maps from E_τ to itself that respect the group structure are those of the form $z \mapsto cz$, with c a complex number such that $c \cdot (\mathbf{Z}+\mathbf{Z}\tau)$ is contained in $\mathbf{Z}+\mathbf{Z}\tau$. All such c are easily seen to be integers, except when the field extension of $\mathbf{Q}$ generated by τ has degree two, in which case E_τ is said to have complex multiplications.

Let now J be the set of all complex numbers of the form $j(\tau)$, with τ quadratic over $\mathbf{Q}$. The aim of the talk is to classify all irreducible polynomials f in $\mathbf{C}[x,y]$ for which there exist infinitely many (a,b) in $J \times J$ with $f(a,b) = 0$. Examples of such f are $x-a$ and $y-b$ for a and b in J. For each poitive integer n one has Kronecker's polynomial F_n in $\mathbf{Z}[x,y]$, characterized (up to sign) by the properties that it is irreducible and satisfies $F_n(j(\tau), j(n\tau)) = 0$ for all τ in the upper half plane, giving some more examples. It will be shown that the generalized Riemann hypothesis implies that these examples are in fact the only ones, up to multiplication by scalars. It will also be explained how this result gives evidence for a very general conjecture made by F. Oort. The proof uses some algebraic number theory, some analytic number theory and some real analytic group theory.