Brian Conrad, Institute for Advanced Study "Deformation theory of Galois representations for the non-expert"

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Our aim is to give some idea (to a non-number theorist) of what Mazur's deformation theory of Galois representations is about and how its applicability to the study of elliptic curves has been extended since Wiles' breakthrough. The Taniyama-Shimura conjecture relates, in a very non-trivial way, the algebraic theory of elliptic curves and the analytic theory of modular forms. We will use Wiles' reformulation of this conjecture in terms of deformation theory as a means of indicating what the deformation theory can do. Extending these methods to more elliptic curves has required some generalizations of the deformation problems considered. We give an overview of some of this work, as well a big remaining deformation-theoretic difficulty.