

Mathematical problems suggested by Analog-to-Digital conversion

Ingrid Daubechies, Princeton University

In Analog-to-Digital conversion, continuously varying functions, (e.g. the output of a microphone) are transformed into digital sequences from which one then hopes to be able to reconstruct a close approximation to the original function. The functions under consideration are typically band-limited (i.e. their Fourier transform is zero for frequencies higher than some given value, called the bandwidth); such functions are completely determined by samples take at a rate determined by their bandwidth. These samples then have to be approximated by a finite binary representation. Surprisingly, in many practical applications, one does not just replace each sample by a truncated binary expansion; for various technical reasons, it is more attractive to sample much more often and to replace each sample by just 1 or -1, chosen judiciously.

In this talk, we shall see what the attractions are of this quantization scheme, and discuss several interesting mathematical questions suggested by their approach. This will be a review of work by many others as well as myself, including Ron DeVore, Sinan Gunturk, Thao Nguyen and Ozgur Yilmaz. It is also a case study of how continuous interaction with engineers helped to shape and change the problems as we tried to make them more precise.