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``Hermitian matrices, tensor products, and honeycombs''

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Given two Hermitian matrices whose eigenvalues we know, what might be the eigenvalues of the sum? In 1912 Weyl gave a short list of necessary conditions, and in 1962 Horn conjectured an explicit list of necessary and sufficient linear inequalities relating the spectra of the inputs to that of the sum. In the early 1990's Klyachko proved a different (and less explicit) list to be necessary and sufficient, but his list is redundant -- some of his inequalities imply others. This problem is also intimately related to the following question: when we tensor together two irreducible representations of $GL(n)$, which irreps occur in the product?

We introduce a new combinatorial object called a honeycomb to study these problems. Using honeycombs, we prove Horn's conjecture, determine which of Klyachko's (and Horn's) conditions are redundant, and show that there are no nasty surprises on going from the Hermitian matrix problem to the tensor product problem. We comment on some other problems that admit easy study via honeycombs.

This work is joint with Terry Tao of UCLA and Chris Woodward of Rutgers and MSRI.