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120 Latimer

Fall 1997, Math 53M
First Midterm Exam

30 Sept., 1997
8:10-9:30 AM

1. (25 points) Let C be the plane curve defined by the parametric equations $x = \sin t$, $y = \tan t$ $(t \in (-\pi/2, \pi/2))$.

(a) (7 points) Sketch the curve C, showing any maxima, minima, intercepts or asymptotes it may have.

(b) (7+11 points) Obtain formulas for dy/dx and d^2y/dx^2 on this curve C as functions of t.

2. (15 points) Compute the arc length of the space curve $\mathbf{r}(t) = (t^2/2 - \ln t)\mathbf{i} + (2 \sin t)\mathbf{j} - (2 \cos t)\mathbf{k}$ from $\mathbf{r}(1)$ to $\mathbf{r}(3)$.

3. (20 points) Suppose F is a differentiable function of two variables, whose domain includes (1,1), and we write $F(1,1) = a$, $F_x(1,1) = b$, $F_y(1,1) = c$.

(a) (12 points) Express in terms of a, b and c the partial derivatives of $(x+y)F(x,y)$ at the point (1,1).

(b) (8 points) Find the directional derivative of $(x+y)F(x,y)$ at the point $(1,1)$ in the direction of the vector $\langle 3,4 \rangle$.

4. (25 points) Suppose F is a differentiable function of three variables, and we define a function G of two variables by $G(x,y) = F(x+y, xy, x)$.

(a) (13 points) Express the partial derivatives of $G(x,y)$ with respect to x and y in terms of F and its partial derivatives F_1, F_2, F_3.

(b) (12 points) Let (a,b) be a point of the plane, and C the level curve to the function $G(x,y)$ (defined above) which passes through (a,b). Find the equation of the tangent line to C at (a,b) in terms of the value and partial derivatives of F.

5. (15 points) Suppose u and v are differentiable functions of two variables. Derive the formula $\nabla(u/v) = (v\nabla u - u\nabla v)/v^2$.