Math 54W, Fall '97, K Miller p1 of 2
Final Exam, Dec 16 Name ________________
Name of TA ____________ Time of TA section ______

1a Give the defn of eigenvalue for an $n \times n$ matrix A.
Give the defn of "the vectors $\underline{v}_1 \ldots \underline{v}_k$ are linearly independent"

1b Find bases for $RS(A)$, $CS(A)$, and $NS(A)$ for

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ -1 & -1 & 0 & -2 \\ 1 & 0 & 1 & 1 \end{pmatrix}.$$

2a Suppose A is symmetric and $\underline{v}_1, \underline{v}_2$ are two eigenvectors of A corresponding to different eigenvalues $\lambda_1 \neq \lambda_2$. Show that $\underline{v}_1$ and $\underline{v}_2$ are orthogonal. Hint: consider $(A\underline{v}_1) \cdot \underline{v}_2$.

2b Find the eigenvalues for $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$ and a basis for each of its eigenspaces. Then find three mutually orthogonal eigenvectors $\underline{v}_1, \underline{v}_2, \underline{v}_3$.

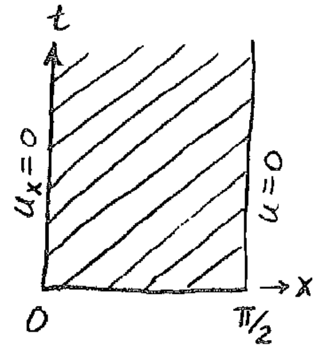
3a Find the general real-valued solution $y(t)$ of the 4th order differential equation $y'''' + 8y'' + 16y = 0$.

3b Find the general real solution $y(t)$ of the 2nd order equation $y'' + \mu^2 y = 0$, where μ is a positive real number. Then find that solution with the initial values $\begin{pmatrix} y(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

<u>4</u> Find the solution $\underline{x}(t)$ of the following initial value problem:

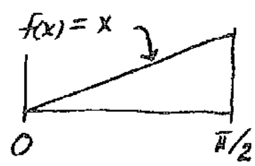
$$\begin{cases} \underline{x}'(t) = \begin{pmatrix} 3 & -5 \\ -1 & -1 \end{pmatrix} \underline{x}(t), \\ \underline{x}(0) = \begin{pmatrix} 6 \\ -6 \end{pmatrix}. \end{cases}$$

<u>5</u> Use separation of variables to find the "special solutions" of the following heat equation and boundary conditions:



① $u_t = u_{xx}$ for $0 < x < \pi/2$, $t > 0$,
② $u_x(0,t) = 0$ for $t > 0$,
③ $u(\pi/2, t) = 0$ for $t > 0$.
(I <u>grant</u> you that the "separation constant" σ is <u>not</u> negative, so you must consider the zero and positive cases. <u>Explain</u> each step of your derivation.)

<u>6a</u> Let $f(x) \equiv x$ for $0 < x < \pi/2$, and suppose that it is possible to write $f(x)$ as an infinite series of the form $f(x) = (c_1 \sin 1x + c_3 \sin 3x + c_5 \sin 5x + \dots)$ for $0 < x < \pi/2$. I <u>grant</u> you that these sine functions S_n ($n = 1, 3, 5, \dots$) are orthogonal on $(0, \pi/2)$ with $(S_n, S_n) = \pi/4$. <u>Find</u> the coefficients c_n.



<u>6b</u> Give the <u>statement</u> of the existence and uniqueness theorem for the <u>linear</u> 1st order initial value problem $\begin{cases} \underline{x}'(t) = P(t)\,\underline{x}(t) + \underline{g}(t) \text{ for } t \text{ in } I, \\ \underline{x}(t_0) = \underline{x}^0. \end{cases}$