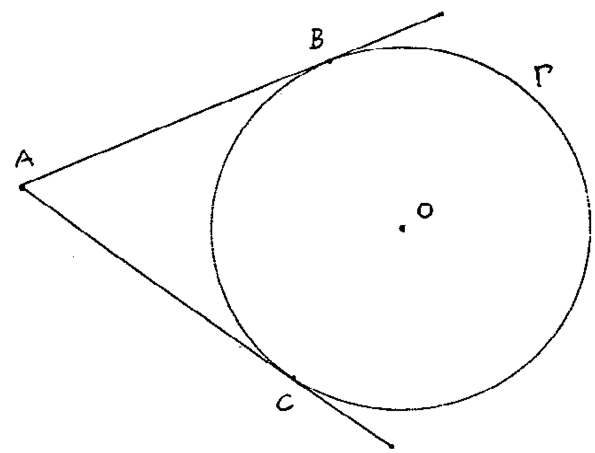
Classical Geometry
Math 130
R. Hartshorne

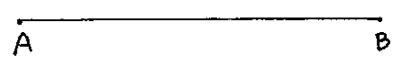
Final Exam, Dec. 11, 1996
Ten questions; ten points each.

1. If A is a point outside a circle Γ, and if AB and AC are two tangents from A to Γ, prove that $AB \cong AC$. Give a proof in the style of Euclid, using only results from Euclid Books I–IV, and refer to each result used by Book and proposition number.

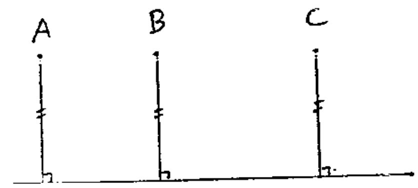


2. Make a ruler and compass construction of a figure with the properties described in Exercise 1.15 (p.14 of notes), in 25 steps or less. List and number your steps one by one, and explain at the side what each set of steps is accomplishing (e.g. "make a perpendicular to line AB at the point B").

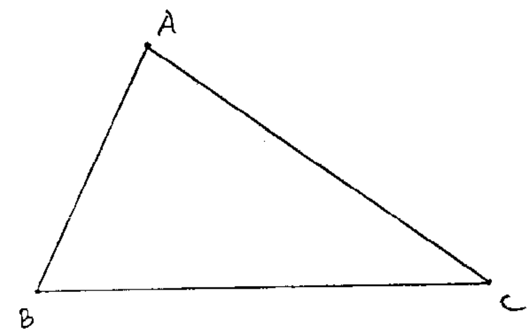
3. Construct with ruler and compass a regular pentagon having the given segment AB as one side, in 20 or fewer steps. List and number your steps as usual, and explain what they are doing.



4. In a Hilbert plane satisfying Playfair's axiom (P), show the following: If ℓ is a line, and if A, B, C are three points on the same side of ℓ, and if the perpendicular segments from A, B, C to ℓ are all equal, then A, B, C lie on a line. In your proof, you may use Hilbert's axioms and results in the text (referred to by section and number).



5. In a Hilbert plane, show that the three angle bisectors of a triangle meet in a point (and include a proof that they actually meet at all). In your proof, you may use Hilbert's axioms and results in the text (referred to by section and number).



6. (a) In your own words, explain the main ideas of Euclid's proof of the construction of the regular pentagon (IV.11). You do not need to give a detailed proof.

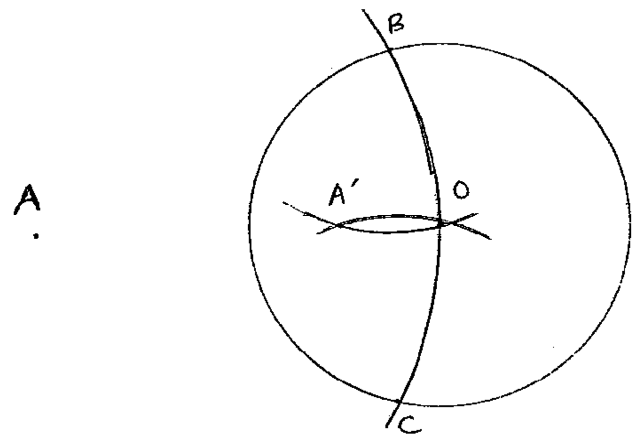
(b) In the Cartesian plane over a field F satisfying $(**)$ (cf. p.118), give a complete proof that the construction of (4.3) on p.45 produces a regular pentagon.

7. In the Poincaré model prove directly that for any P-point A, and any P-line ℓ not containing A, there is a P-line m through A and meeting ℓ perpendicularly.

(a) First translate the statement into Euclidean geometry.

(b) Then prove that statement using Euclidean geometry in the ambient Cartesian plane.

8. In the Cartesian plane over a field F satisfying $(**)$, prove that the following construction, where A is a point outside the circle Γ, gives the circular inverse of A: Draw a circle with center A and radius AO, where O is the center of Γ. Let it meet Γ at B and C. Let the circles BO and CO meet at A'. Then A' is the circular inverse of A with respect to Γ.



9. In a hyperbolic plane, let ℓ and m be limiting parallel lines. Prove that there exists a line n which is limiting parallel to ℓ at the other end of ℓ, and meets m at a right angle. You may quote results in the text by number.

10. In the hyperbolic plane, let F be Hilbert's field of ends. Let O = point where line $(0, \infty)$ meets the line $(1, -1)$ and let A = point where the line $(1, -1)$ meets the line (a, a^{-1}), where $a \in F$, $a > 1$. Explain the meaning of the multiplicative distance function μ, and find $\mu(OA)$ as an element of F.